Dual approach to growth accounting – application for the Czech Republic

Miroslav Hloušek

Faculty of Economics and Administration of Masaryk University,
Department of Applied Mathematics and Computer Science,
Lipová 41a, 602 00 Brno
e-mail: hlousek@econ.muni.cz

Abstract

This paper tries to compare results from primal and dual approach to growth accounting for case of the Czech Republic. The dual approach is based on factor prices rather than quantities and hence the result should be more reliable. Using both approaches, total factor productivity growth rate is calculated from Czech data. Results are quite similar which suggests that national accounts provided by statistical agency are correct and that dual approach is useful alternative to growth accounting. Additionally, TFP growth accounts for seventy seven per cent of the growth rate of total output and plays important role in explaining growth.

Keywords

growth accounting, primal and dual approach, total factor productivity growth rate, Solow residual

1 Introduction

Growth accounting is decomposition of the growth rate of output into contributions of individual factors of production. The main point is to find out if factor accumulation or "something else" plays crucial role. The something else is usually called Total Factor Productivity (TFP) or Solow residual and measures technology progress.

The first exercise of growth accounting was presented by Solow (1957) and was based on subtracting of weighted growth rates of capital and labor from the growth rate of output. The remaining (residual) part was ascribed to growth of technology. This is primal approach to growth accounting. The main problem of primal approach is that it relies on measuring inputs such as capital or labor which can be sometimes unreliable. On the other hand, dual approach is based on factor prices rather than quantities, which are usually better to measure. Next advantage of price-based estimates is that the prices are formed at the markets and agents have incentive to get the right price. A firm that pays to factor more than its marginal product is throwing money away. On the contrary, national accounts are provided by government (statistical) agencies and the figures may be subject to large errors.

This paper exploits the advantage of dual approach and tries to compare results from both approaches on Czech data. The paper is organized as follows. Derivation of TFP growth rate from factor prices is presented in Section 2. Section 3 briefly describes data and their transformation. Section 4 compares results from dual and primal approach and Section 5 makes summary and concludes with prospects for further research. Appendix deals with primal approach, both theoretically and empirically.

2 Dual approach to growth accounting

The first exposition of equivalence of primal and dual approach was presented by Jorgenson and Griliches (1967). Here, I follow more transparent explanation provided by Hsieh (2002). He starts with national income accounting identity that output is equal to factor incomes

$$Y = rK + wL \tag{1}$$

where Y is output, K is capital, L is labor and r and w is the rental price of capital and the real wage respectively.

Differentiation of (1) with respect to time and dividing by Y gives

$$\frac{\dot{Y}}{Y} = s_K \left(\frac{\dot{r}}{r} + \frac{\dot{K}}{K} \right) + s_L \left(\frac{\dot{w}}{w} + \frac{\dot{L}}{L} \right) \tag{2}$$

where $s_K = rK/Y$ and $s_L = wL/Y$ are the factor-income shares.[1] Rewriting into more convenient way where variables with "hat" denotes growth rates gives

$$\hat{Y} = s_K(\hat{r} + \hat{K}) + s_L(\hat{w} + \hat{L}). \tag{3}$$

Placing terms of the growth rates of factor production on left-hand side of the equation, one obtains

$$\hat{Y} - s_K\hat{K} - s_L\hat{L} = s_K\hat{r} + s_L\hat{w}. \tag{4}$$

The left-hand side of equation (4) is primal estimate of Solow residual or Total Factor Productivity growth. Share-weighted growth rates in factor quantities are subtracted from the growth rate of output

$$\text{TFP}_{\text{primal}} = \hat{Y} - s_K\hat{K} - s_L\hat{L}. \tag{5}$$

The right-hand side of equation (4) is dual measure of TFP that is obtained as share-weighted growth in factor prices

$$\text{TFP}_{\text{dual}} = s_K\hat{r} + s_L\hat{w}. \tag{6}$$

The primal and dual measures of TFP growth rate should be the same with only the condition that output equals factor incomes.[2] No other assumptions about the form of the production function, bias of technological change or relationship between factor prices and their social marginal products need to be made.[3] The two measures will differ only if the national accounts are inconsistent with the data on factor prices.

3 Data

The growth accounting exercise can be easily extended to allow for different types of capital and labor. However, this paper deals only with aggregate measures of factor inputs (and their prices). Data are obtained from databases of Czech Statistical Office and Czech National Bank. Frequency of data is quarterly, sample period is from 1996Q1

---

[1] In this setting, the sum of factor-shares is equal to unit, $s_K + s_L = 1$

[2] Equation (1)

[3] If factor prices deviate from social marginal products, the estimated value of TFP would deviate from the "true" value. However, the error from dual approach will be the same as that from the primal approach.

to 2006Q4. Specifically, the real interest rate that measures the rental price of capital is represented by 3 month nominal PRIBOR deflated by inflation (ex ante approach).[4] The real wage is calculated as a ratio of nominal wage rate and consumer price index. These time series are plotted in Figure 1.
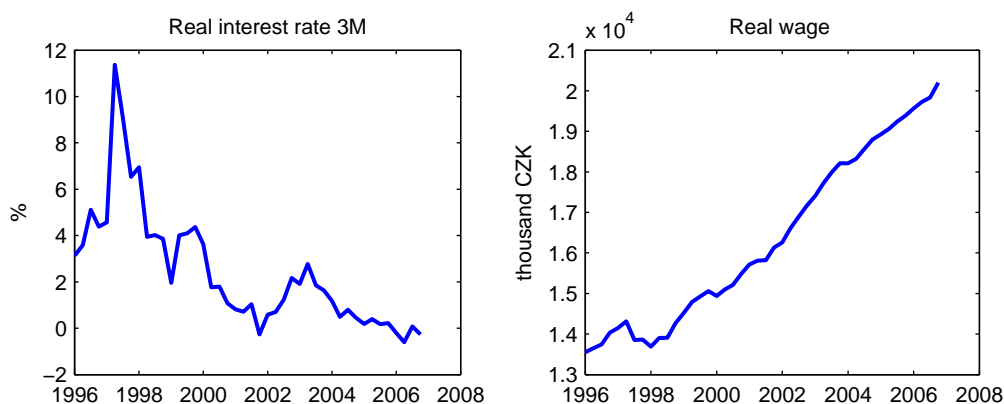

Figure 1: Prices of factors of production

To calculate the TFP growth one also needs factor-income shares. The labor-income share is obtained from annual data of gross value added (current prices), nominal costs of labor per person (including social expenditures) and number of employed persons. The ratio of total cost of labor to gross value added which corresponds to $s_L = wL/Y$ is calculated. Subsequently capital-income share s_K is computed as complement to unit $s_K = 1 - s_L$. The time span covers years from 1996 to 2005. The average share of labor s_L in this period is 59 %.

4   Results

Calculation of the growth rates of factor prices and TFP using equation (6) is now easy task. Graphical results (growth rates of factor prices and TFP) are presented in Figure 2, numerical results can be seen in Table 1. For better comparison with primal approach, the results in the table relates to period from 1996Q1 to 2005Q4. As I used quarterly data, the growth rates are expressed in both quarterly and annualized expression. The last two rows shows annualized weighted growth rates of factor prices (using factor-income shares) where the last row includes the whole sample period (1996Q1 – 2006Q4).

The rental price of capital decreased on average during the time which corresponds to decreasing marginal product of capital associated with growing capital stock. On the other hand, the real wage and thus marginal productivity of labor was increasing as employment decreased.[5] Comparison of TFP growth rates measured by dual and primal approach (last two columns) is quite satisfactory. Both estimates are very similar, the difference is only 0.08 percentage points for annual expression. These results suggest that national accounts provided by statistical office are consistent with factor prices. The

---

[4]Actually, the rental price of capital is the real interest rate plus depreciation rate. But assuming constant depreciation rate during the time, the growth rate of the rental price will be the same as the growth rate of the real interest rate.

[5]See corresponding results from primal approach in Table 2 in Appendix.

dual approach provides good alternative of measuring and verifying TFP growth. Its main advantage lies in availability of data – time series are registered quarterly and are accessible soon.

In addition, TFP growth rate plays central role in explaining growth in the Czech Republic. It accounts for nearly 77 % of the growth rate of output.[6] This result supports findings of Easterly and Levine (2001). "Something else" and not factor accumulation is critical for understanding economic growth and theories of growth should be aimed at modeling and quantifying TFP growth.
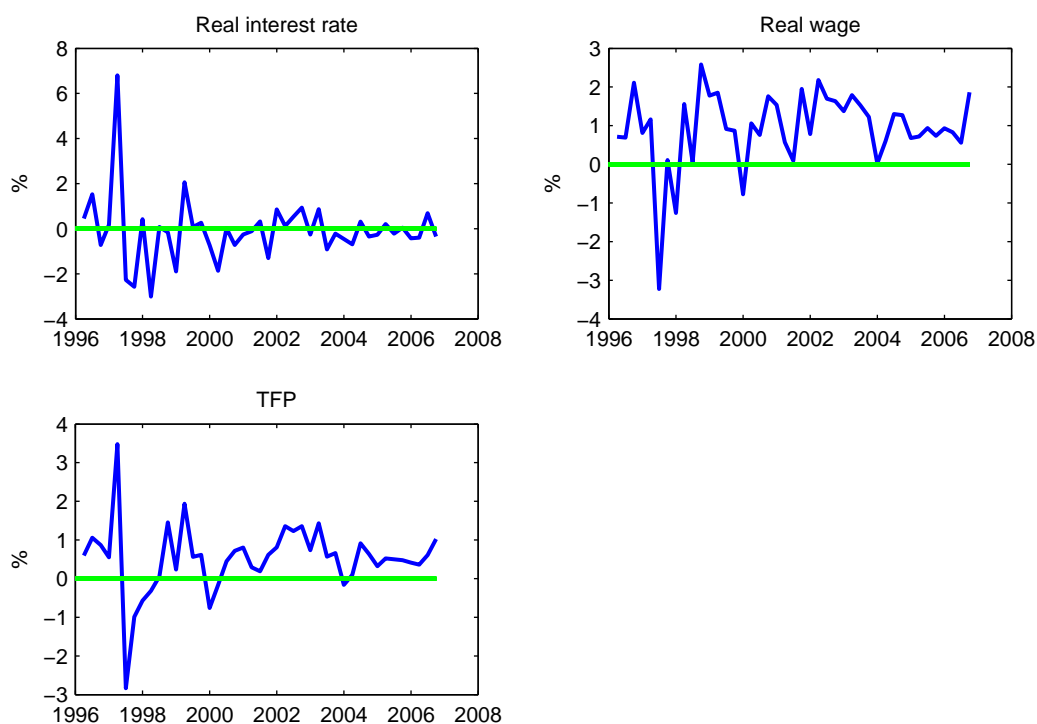

Figure 2: Growth rates

	Rental price of capital	Real Wage	Dual TFP	Primal TFP
Quarterly	-0.07	0.92	0.52	0.49
Annual	-0.30	3.67	2.06	1.98
Annual weighted	-0.12	2.18	2.06	1.98
Annual weighted*	-0.13	2.23	2.11	n.a.

Table 1: Dual approach (growth rates in %)

---

[6]See again Table 2 in Appendix.
*Results from period 1996Q1 – 2006Q4.

5 Conclusion

This paper examined two approaches to growth accounting: *primal* that is based on quantities of factor inputs and *dual* that is based on factor prices. The analysis used Czech time series of aggregate variables. The results of the exercise are quite satisfactory. Both estimates of TFP growth are very similar and dual approach is useful alternative to measuring TFP. Next, the results point out that total factor productivity (and not factor accumulation) is more important for understanding economic growth, at least in case of the Czech Republic. These findings should be taken into consideration for formulation of theory and policy of economic growth.

The analysis can be further extended to more detailed specification of factors of production (such as various types of labor or capital) and their corresponding prices. Further research will be also focused on other countries and cross-country comparisons of TFP growth rates.

6 Appendix

Derivation of TFP growth (or Solow residual) using primal approach is taken from Barro and Sala-i-Martin (2004). Consider general production function

$$Y = F(T, K, L),$$

where T is level of technology, K is capital and L is labor. Taking logarithm and derivatives with respect to time we get[7]

$$\frac{\dot{Y}}{Y} = \frac{F_T T}{Y} \frac{\dot{T}}{T} + \frac{F_K K}{Y} \frac{\dot{K}}{K} + \frac{F_L L}{Y} \frac{\dot{L}}{L}, \tag{7}$$

where F_K and F_L are social marginal products of capital and labor respectively. The contribution of technological progress to growth is

$$g = \frac{F_T T}{Y} \frac{\dot{T}}{T}$$

which can be obtained as subtraction of (weighted) growth rates of individual inputs from the growth rate of output

$$g = \frac{\dot{Y}}{Y} - \frac{F_K K}{Y} \frac{\dot{K}}{K} - \frac{F_L L}{Y} \frac{\dot{L}}{L}. \tag{8}$$

For empirical analysis, I use production function with Hicks-neutral technological progress: $F(T, K, L) = T\tilde{F}(K, L)$ which gives $F_T T = Y$ and $g = \dot{T}/T$.[8]

If the factors are paid their social marginal products then, with regard to previous notation, $F_K K/Y$ and $F_L L/Y$ are equal to factor shares of capital and labor on total income $s_K = rK/Y$ and $s_L = wL/Y$. Then equation (8) becomes familiar equation (5).

$$g = \text{TFP}_{\text{primal}} = \frac{\dot{Y}}{Y} - s_K \frac{\dot{K}}{K} - s_L \frac{\dot{L}}{L}.$$

---

[7]F_X denotes derivative of the function with respect to variable X.

[8]The way of introducing of technological progress into production function has not important influence for results.

Data used for calculation of primal TFP growth are obtained from Czech Statistical Office. Data are annual, from period 1995 to 2005. Specifically, I use gross domestic product (constant prices of 2000), number of employed people (recalculated as equivalent to full-time job) and stock of gross fixed capital (constant prices of 2000). Factor-income shares are the same as for dual approach. The results are presented in Table 2. The first row shows annual growth rates of output and factors or production, the second row represents contribution from individual inputs, i.e. growth rates multiplied by factor-income shares. The last row expresses percentage contribution of growth of each factor to the growth rate of output.

	Output	Capital	Labor	TFP
Annual	2.58	1.90	−0.28	1.98
Annual weighted	2.58	0.76	−0.16	1.98
Contribution to Output	100.00	29.43	−6.12	76.69

Table 2: Primal approach (growth rates in %)

References

[1] ADDERLEY, C. *Somethin' Else,* Blue Note, 1958.

[2] BARRO, R. J. – SALA-I-MARTIN X. *Economic Growth,* Cambridge, MA: MIT Press, second edition, 2004.

[3] EASTERLY, W. – LEVINE, R. Its Not Factor Accumulation: Stylized Facts and Growth Models *World Bank Economic Review,* Vol. 15, No. 2, 2001, pp. 177-219.

[4] HÁJEK, M. Economic growth and total factor productivity in the Czech Republic from 1992 to 2004, *Working Paper CES VŠEM* No. 5, 2005.

[5] HSIEH, C. T. Productivity Growth and Factor Prices in East Asia, *American Economic Review,* Vol. 89, No. 2, May 1999, pp. 133-138.

[6] HURNÍK, J. Potential Output: What Can the Production Function Approach Tell Us? Czech National Bank, Economic Research Bulletin, No. 1, Vol. 3, May 2005, pp. 5-7.

[7] JORGENSON D. W. – GRILICHES Z. The Explanation of Productivity Change, *The Review of Economic Studies,* Vol. 34, No. 3, July, 1967, pp. 249-283.

[8] SOLOW, R.M. Technical Change and the Aggregate Production Function, *The Review of Economics and Statistics,* Vol. 39, No. 3, August, 1957, pp. 312-320.