Business cycles

Deterministic view of business cycles (fluctuations around trend) is little bit old fashioned. Cycles produced by deterministic models are too regular to be consistent with stylized facts.

A more plausible explanation is **stochastic** approach. It supposes that an economy is constantly subject to random disturbances, or shocks. These might originate in *demand* side (change in consumer preferences, actions of government) as well in *supply* side (bad or good crops, important inventions or discoveries – technology inovations). The list of potential shocks is endless.

These shocks are often referred to as **impulses**. They hit the economic system, which reacts by generating business cycles. The cycles result from averaging or cumulation of these random distrubances over time. The process that transforms impulses into oscillations is called **propagation mechanism**.

However there exists some systematic mechanisms in the economy that ensure the return to the stationary state. They are engaged all the time, but new and new shocks hit the economic system and it never really settles down to its stationary state. So we observe oscillations – business cycles – that result from random disturbances.

Example

We assume that the cyclical behavior of gap of the real GDP can be decribed by simple stationary autoregressive process AR(1).

$$\hat{y}_t = \alpha \hat{y}_{t-1} + \epsilon_t, \tag{1}$$

where $\hat{y}_t$ is gap of GDP, α is autoregressive parameter: $\alpha \in (0,1)$, ϵ_t is random shock in business cycle with these properties: $\epsilon_t \sim WN(0, \sigma^2)$.

This equation is only one possible "candidate" for description of behaviour of GDP gap; it is not complete explanation of cyclical fluctuations.[1] It is example of *model in reduced form*; we don't know the behavioural foundations of parameter α.

The systematic part of behaviour of output gap is the autoregressive component:

$$\hat{y}_t = \alpha \hat{y}_{t-1},$$

The stochastic element is the cyclical shock ϵ_t.

[1] It is interesting fact that this equation is able to replicate basic behaviour of real data even in this simple form. Compare simulated process with estimation of GDP gap of the United States.

Assume that cyclical shock of magnitude ϵ hits the economy in time $t = 0$. Its contribution to current and future values of output gap will be:

$$\epsilon_0,\ \alpha\epsilon_0,\ \alpha^2\epsilon_0,\ \alpha^3\epsilon_0,\ \ldots, \alpha^n\,\epsilon_0$$

If $\alpha \in (0,1)$ the influence of shock is decreasing over time. For $n \to \infty$ the shock disappears and has no permanent efect in this process. We can interpret it that, in absence of other shocks, the economic mechanisms ensure return of output to its trend value (closing of output gap).

Application

Using OLS method we can estimate parameter α of the process (1) on real data: GDP gap of USA. A possible interpretation of parameter α in this model is that it expresses degree of persistence (or inversely degree of flexibility) of economy in reaction to shocks. The larger is this parameter, the longer is the influence of the cyclical shock.

We can calculate point of time when 90 % of the shock has disappeared (or equivalently, when the shock is smaller than 10 % of its original magnitude). The influence of shock in time n is

$$\alpha^n\epsilon$$

We want to find time n for that holds:

$$\alpha^n\epsilon \leq (1-0.90)\epsilon \quad \Rightarrow \quad n \geq \frac{\log 0.10}{\log \alpha}$$

Using standard deviation of residuals and parameter α we can also compute standard deviation of the output gap in this model. It expresses "average" value how much the actual GDP deviates from its trend value.

$$\text{var}\ \hat{y}_t = \frac{\text{var}\ \epsilon_t}{1-\alpha^2} \quad \Rightarrow \quad \text{std}\ \hat{y}_t = \sqrt{\frac{(\text{std}\ \epsilon_t)^2}{1-\alpha^2}},$$

where 'var' denotes variance and 'std' denotes standard deviation.

Futher readings

Burda, M., Wyplozs, C. *Macroeconomics: A European Text* New York: Oxford University Press, 2001, Chapter 14, Section 14.3

Appendix: more detailed derivations

Shock assumed only in $t = 0$, ϵ_0, otherwise 0. Output gap before shock is also 0, $\hat{y}_{-1}, \hat{y}_{-2}, \ldots = 0$

$$\begin{aligned}
\hat{y}_0 &= \alpha\hat{y}_{-1} + \epsilon_0 = \epsilon_0 \\
\hat{y}_1 &= \alpha\hat{y}_0 + \epsilon_1 = \alpha\epsilon_0 \\
\hat{y}_2 &= \alpha\hat{y}_1 = \alpha\alpha\epsilon_0 = \alpha^2\epsilon_0 \\
\hat{y}_3 &= \alpha\hat{y}_2 = \alpha^3\epsilon_0 \\
&\vdots \\
\hat{y}_n &= \alpha^n\epsilon_0
\end{aligned}$$

$$\lim_{n\to\infty} \alpha^n\epsilon_0 = 0$$

Time period when the shock is less than 10 % of its original value.

$$\begin{aligned}
\alpha^n\epsilon &\leq 0.1\epsilon \\
\log\alpha^n &\leq \log 0.1 \\
n\log\alpha &\leq \log 0.1 \\
\\
n &\geq \frac{\log 0.1}{\log\alpha}
\end{aligned}$$

Standard deviation of GDP gap for AR(1) model.

$$\begin{aligned}
\hat{y}_t &= \alpha\hat{y}_{t-1} + \epsilon_t \\
\mathrm{var}(\hat{y}_t) &= \mathrm{var}(\alpha\hat{y}_{t-1} + \epsilon_t) \\
\mathrm{var}(\hat{y}_t) &= \mathrm{var}(\alpha\hat{y}_{t-1}) + \mathrm{var}(\epsilon_t) + 2\mathrm{cov}(\alpha\hat{y}_t, \epsilon_t) \\
\mathrm{var}(\hat{y}_t) &= \alpha^2\mathrm{var}(\hat{y}_t) + \mathrm{var}(\epsilon_t) \\
(1-\alpha^2)\mathrm{var}(\hat{y}_t) &= \mathrm{var}(\epsilon_t) \\
\\
\mathrm{var}(\hat{y}_t) &= \frac{\mathrm{var}(\epsilon_t)}{1-\alpha^2}
\end{aligned}$$