Testing of Long-run Relationships

Analysis of trends or non-stationary time series is more complicated. If we want to test such long-run relationships, it is necessary to use complex tools of cointegration analysis. However, we will skip formal testing of conitegration relationship and come directly to quite simple technique – error-correction model.

Example: We want to test following relationship (simplified form of money demand)

$$m_t = \alpha p_t + \beta y_t, \tag{1}$$

where m_t is logarithm of money stock, p_t is logarithm of price level and y_t is logarithm of real output. These time series can be non-stationary.

If we are interested in long-run relationship between trends of these variables, we assume that this equation is not valid in every period of time, but in "average" in long time horizon. Formally, we can express deviations from the equation (1) as

$$\psi_t = m_t - \alpha p_t - \beta y_t. \tag{2}$$

where ψ_t is already stationary time series. The deviations fluctuate around zero, they are only transitory and in the long-run they disappear. In other words, there exist some economic mechanisms that are able to correct any deviation from long-run relationship (1).

Additionally, we have to assume that the deviations are corrected by only **one** of the three variables, for example m_t. Price level and real output are in this case *weakly exogenous*. If any deviation appears, change of money stock (not change of price level or real output) will correct it.

Then we can estimate equation of this form

$$\Delta m_t = \gamma(m_{t-1} - \alpha p_{t-1} - \beta y_{t-1}) + \ldots + \epsilon_t \tag{3}$$

where γ is negative parameter (if estimated value of this parameter comes out positive, there can't be long-run relationship) and instead of dots ... other explaining variables can be added. Lagged differences (Δm_{t-1}, Δm_{t-2} or Δp_{t-1}, Δy_{t-1}, etc.) are usually used. These additional variables are not so important for long-run relationship, they only improve statistic properties of the model.

Important point is that there are only stationary variables in the equation (3). Estimation of the parameters of such relationship is possible using classical tools (ordinary least squares method).

Interpretation of the equation (3) is as follows: if there is, for example, positive deviation from the long-run relationship in time $t-1$

$$m_{t-1} - \alpha p_{t-1} - \beta y_{t-1} > 0$$

it means that m_{t-1} is "too high" (too much money in circulation) in relation to given p_{t-1} and y_{t-1}. This deviation is multiplied by negative parameter γ and it will cause decrease of money Δm_t in the following period. The deviation is corrected toward establishing of long-run relationship. (If negative deviation appears the process of correction is inverse.)