Growth Accounting

Principle of growth accounting is decomposition of the growth rate of aggregate output into the growth rates of individual inputs, usually capital, labor, technology. The analysis starts from a standard neoclassical production function

$$Y_t = F(A_t, K_t, L_t)$$

where Y_t is output,K_t and L_t are capital and labor respectively and A_t is level of technology.

The concrete form of the production function is:

$$Y_t = K_t^{1-\alpha_t}(A_t L_t)^{\alpha_t}$$

The technology progress is assumed to be labor-augmenting, α_t is labor intensity of production, and is restricted as $\alpha \in (0,1)$. This parameter can be variable during the time.

We assume that remunerations to the factors of production are determined by their marginal products. Marginal products are derivatives with respect to labor and capital.

$$\frac{\partial F(...)}{\partial L_t} = \alpha_t K_t^{1-\alpha_t} A_t^{\alpha} L_t^{\alpha_t - 1} = \alpha_t \frac{Y_t}{L_t} = W_t$$

$$\frac{\partial F(...)}{\partial K_t} = (1-\alpha_t)\frac{Y_t}{K_t} = R_t$$

where W_t is the real wage and R_t is the real interest rate.

The equality of the marginal product of labor and the real wage implies

$$\alpha_t = \frac{W_t L_t}{Y_t}$$

It shows that the unobservable parameter in the production function can be estimated as the share of wage payments to labor over total income.

The growth of aggregate output can be decomposed using total differential

$$\frac{dY_t}{Y_t} = \frac{1}{Y_t}\frac{\partial F(...)}{\partial K_t}dK_t + \frac{1}{Y_t}\frac{\partial F(...)}{\partial A_t}dA_t + \frac{1}{Y_t}\frac{\partial F(...)}{\partial L_t}dL_t$$

After inserting equations used above we get

$$\frac{dY_t}{Y_t} = (1-\alpha_t)\frac{dK_t}{K_t} + \alpha_t\frac{dA_t}{A_t} + \alpha_t\frac{dL_t}{L_t}$$

The continuous-time formula can be modified for empirical purposes to apply to discrete time using approximation

$$\frac{dY_t}{Y_t} \approx \frac{Y_t - Y_{t-1}}{Y_{t-1}} = \frac{\Delta Y_t}{Y_{t-1}}$$

$$\frac{\Delta Y_t}{Y_{t-1}} = (1-\alpha_t)\frac{\Delta K_t}{K_{t-1}} + \alpha_t\frac{\Delta A_t}{A_{t-1}} + \alpha_t\frac{\Delta L_t}{L_{t-1}} \tag{1}$$

The only term in equation (1) that cannot be measured directly is the growth rate of technology

$$\frac{\Delta A_t}{A_{t-1}}$$

However it can be obtained indirectly: we subtract from $\Delta Y_t/Y_{t-1}$ the part of the growth rate that can be accounted for by the growth rate of the inputs, K_t and L_t. The part that remains, which provides an estimate of the growth rate of A_t, is often called Solow residual or Total Factor Productivity (TFP).

Which factors lead to changes in TFP? Essentialy, any change implying that an economy can produce more aggregate output with the same factor inputs is an increase in total factor productivity. Factors that increase TFP include e.g. good weather, technological innovations, the easing of govenrment regulations or decreases in the relative price of energy.

Limitations

- Quality of inputs and output was not taken into consideration.
- Assumption of perfect competition (Factors of production are paid their marginal product).
- Growth accounting – mechanical decomposition, not theory of growth.