1. PODKLAD K PŘEDNÁŠCE – KONVERGENCE BSIM 1.2.13

C-D case

$$s \cdot k_*^{\alpha} = (n+\delta+x)\, k_*$$ s.s.

fundamental eq.

$$\dot{k} = s \cdot k^{\alpha} - (n+\delta+x)k$$

$$s = (n+\delta+x)\, k_*^{1-\alpha}$$

$$\frac{\dot{k}}{k} = s \cdot \frac{k^{\alpha}}{k} - (n+\delta+x)$$

$$\frac{\dot{k}}{k} = (n+\delta+x) \cdot k_*^{1-\alpha} \cdot k^{\alpha-1} - (n+\delta+x)$$

$$\underbrace{\frac{\dot{k}}{k} = (n+\delta+x)\left[\left(\frac{k}{k_*}\right)^{\alpha-1} - 1\right]}$$

linearizace pro Taylora

$$\frac{\dot{k}}{k} = \beta(\ln k^* - \ln k)$$

$$\beta = (1-\alpha)(n+x+\delta)$$

rychlost konvergence

$$\beta = -\frac{\partial(\dot{k}/k)}{\partial \log k}$$ – jak moc se změní (1) tempo růstu, když se změní kapitálová zásoba (1)

$$\frac{\dot{k}}{k} = s \cdot \frac{k^{\alpha}}{k} - (x+n+\delta)$$

$$k = e^{\log(k)}$$

$$\frac{\dot{k}}{k} = s \cdot k^{-(1-\alpha)} - (x+n+\delta)$$

$$k^{-(1-\alpha)} = e^{\log(k)^{-(1-\alpha)}} = e^{-(1-\alpha)\log(k)}$$

$$\frac{\dot{k}}{k} = s \cdot e^{-(1-\alpha)\log(k)} - (x+n+\delta)$$

$$\frac{\partial(\dot{k}/k)}{\partial \log k} \Rightarrow s \cdot -(1-\alpha) \cdot e^{-(1-\alpha)\log k} = -(1-\alpha)\cdot s \cdot k^{-(1-\alpha)} = -\beta$$

blízko ss $s = (n+\delta+x)\, k_*^{1-\alpha}$

$$\underline{\beta = (1-\alpha)\cdot s \cdot k^{-(1-\alpha)}}$$

$$\underline{\beta = (1-\alpha)(n+\delta+x)}$$

(2) Taylor expansion

$$f(x) = f(x^*) + \frac{\partial f}{\partial x}\Big|_{x^*}(x-x^*) + \frac{1}{2!}\frac{\partial f}{\partial^2 x}\Big|_{x^*}(x-x^*) + \ldots$$

$$\frac{\dot{k}}{k} = \frac{\partial \log(k)}{\partial t} =$$

$$s \cdot e^{-(1-\alpha)\log k} - (n+x+\delta)$$

$$\frac{\partial \log k}{dt} \approx \underbrace{\overbrace{(n+\delta+x)\cancel{k^{1-\alpha}}}^{s} \cdot \cancel{k^{-(1-\alpha)}} - (n+x+\delta)}_{0} +$$

$$+ \underbrace{s}_{(n+\delta+x)\cancel{k^{1-\alpha}}} \cdot -1(1-\alpha) \cdot k^{-(1-\alpha)} (\log k - \log k^*) + \ldots$$

$$-(1-\alpha) \cdot k^{-(1-\alpha)} (\log k - \log k^*)$$

$$-(1-\alpha)(n+\delta+x)[\log k - \log k^*]$$

$$\frac{\dot{k}}{k} = -\beta(\log k - \log x^*)$$

$$\underline{\beta[\log k^* - \log k]}$$

$$\gamma_{x^\alpha} = \alpha \cdot \gamma_x$$

$$\frac{\dot{y}}{y} = \alpha \cdot \frac{\dot{k}}{k} \qquad y = k^\alpha$$

$$\frac{\dot{k}}{k} \approx -\beta^*(\log k/k^*) \qquad \log\left(\frac{y}{y^*}\right) = \alpha \cdot \log\left(\frac{k}{k^*}\right)$$

$$\frac{1}{\alpha}\log\left(\frac{y}{y^*}\right) = \log\left(\frac{k}{k^*}\right)$$

$$\approx -\beta^*\left(\frac{1}{\alpha}\log\frac{y}{y^*}\right)$$

$$\frac{\dot{y}}{y} = \alpha \cdot (-\beta)^* \frac{1}{\alpha}\log\frac{y}{y^*} = \underline{-\beta^*\log\frac{y}{y^*}}$$

③

$$\frac{\dot{y}}{y} = \alpha \frac{\dot{k}}{k} = \alpha \left[s \cdot k^{-(1-\alpha)} - (x+n+\delta) \right] =$$

$y = k^{\alpha}$

$y^{\frac{1}{\alpha}} = k$

$$= \alpha \left[s \cdot y^{-\frac{(1-\alpha)}{\alpha}} - (x+n+\delta) \right]$$

$s \cdot k^{\alpha} = (\ldots) k$

$s \cdot y = (\ldots) y^{\frac{1}{\alpha}}$

$y^{1-\cancel{\frac{1}{\alpha}}} = \frac{(\ldots)}{s}$

$y^{\frac{\alpha-1}{\alpha}} = \frac{(\ldots)}{s}$

$\frac{(\ldots)}{y^{\frac{\alpha-1}{\alpha}}} = s$

$$\underline{s = y_*^{\frac{1-\alpha}{\alpha}} (n+x+\delta)}$$

$$= \alpha \left[y_*^{\frac{1-\alpha}{\alpha}} (n+x+\delta) \cdot y^{-\frac{(1-\alpha)}{\alpha}} - (x+n+\delta) \right]$$

$$= \alpha (n+x+\delta) \left[\left(\frac{y}{y^*} \right)^{-\frac{(1-\alpha)}{\alpha}} - 1 \right]$$

roof $\beta = - \frac{d \dot{y}/y}{d \log y}$

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$$\underline{= (1-\alpha)(n+x+\delta) \left(y/y^* \right)^{-\frac{1-\alpha}{\alpha}}}$$