Macroeconomic model

Let's assume New-Keynesian model of closed economy that consists of three equations and one identity. The equations can be derived from microfoundations but we will skip this part. It is a gap model, all variables are expressed as a deviation from their equilibrium value.[1]

$$y_t = E_t y_{t+1} + \beta r_t + \epsilon_t^1 \tag{1}$$

$$\pi_t = \gamma E_t \pi_{t+1} + \delta y_t + \epsilon_t^2 \tag{2}$$

$$i_t = \kappa \pi_t + \lambda y_t + \epsilon_t^3 \tag{3}$$

$$r_t = i_t - E_t \pi_{t+1} \tag{4}$$

where y_t denotes output, r_t is the real interest rate, i_t is nominal interest rate and π_t is rate of inflation.

The model contains expected values. Solving of rational expectations models is rather complicated. For empirical analysis so we will use easier way and transform the model into the form with only contemporary and one period lagged variables (i.e. in time t and $t-1$). Another reasoning is that lagged values improve empirical fit.

$$y_t = \alpha y_{t-1} + \beta r_{t-1} + \epsilon_t^1 \tag{5}$$

$$\pi_t = \gamma \pi_{t-1} + \delta y_t + \epsilon_t^2 \tag{6}$$

$$i_t = \omega i_{t-1} + \kappa \pi_t + \lambda y_t + \epsilon_t^3 \tag{7}$$

$$r_{t-1} = i_{t-1} - \pi_t \tag{8}$$

The parameters have following properties: $\alpha \in (0,1)$, $\beta < 0$, $\gamma \in (0,1)$, $\delta > 0$, $\omega \in (0,1)$, $\kappa > 0$, $\lambda > 0$ and $\kappa > 1$. Finally, ϵ_t^j represents exogenous shocks.

The equation (5) corresponds to the aggregate demand equation; it relates aggregate spending to lagged values of output (habbit in consumption) and and negatively to the real interest rate. The equation (6) is an inflation-adjustment equation in which current inflation depends on lagged value of inflation and output. This equation is aggregate supply or so called Phillips curve. The equation (7) is a monetary rule; the nominal interest rate (instrument of monetary policy) depends on lagged nominal interest rate (interest rate smoothing) and on current output and inflation rate. The equation (8) is an identity for the real interest rate. There occur three types of shocks: aggregate demand shock, aggregate supply shock and monetary policy shock.

[1] For simplicity the variables are referred to without attribute "gap" in further text.

We can insert the identity for the real interest rate (8) into aggregate demand equation (5) to simplify the model.

$$
\begin{aligned}
y_t &= \alpha y_{t-1} + \beta(i_{t-1} - \pi_t) + \epsilon_t^1 \\
\pi_t &= \gamma \pi_{t-1} + \delta y_t + \epsilon_t^2 \\
i_t &= \omega i_{t-1} + \kappa \pi_t + \lambda y_t + \epsilon_t^3
\end{aligned}
$$

Empirical counterparts to model variables are gaps of real GDP, nominal interest rate and inflation rate estimated by HP filer. The parameters of individual equations can be estimated using OLS method. Then the equations are converted into the matrix form of VAR model (Vector AutoRegression).

$$A\,x_t = B\,x_{t-1} + \epsilon_t$$

where x_t is vector of variables in time t, concretely $x_t = [y_t, \pi_t, i_t]'$, x_{t-1} is vector of lagged variables, ϵ_t is vector of shocks, A and B are matrixes of estimated parameters. The conversion into matrix form is outlined here.

$$
\begin{array}{ccccccccc}
y_t & +\beta\pi_t & & = & \alpha y_{t-1} & & +\beta i_{t-1} & +\epsilon_t^1 \\
-\delta y_t & +\pi_t & & = & & \gamma\pi_{t-1} & & +\epsilon_t^2 \\
-\lambda y_t & -\kappa\pi_t & +i_t & = & & & \omega i_{t-1} & +\epsilon_t^3
\end{array}
$$

$$
A = \begin{bmatrix} 1 & \beta & 0 \\ -\delta & 1 & 0 \\ -\lambda & -\kappa & 1 \end{bmatrix} \qquad B = \begin{bmatrix} \alpha & 0 & \beta \\ 0 & \gamma & 0 \\ 0 & 0 & \omega \end{bmatrix}
$$

To solve the system, we premultiply both sides by inverse matrix of A.

$$
\begin{aligned}
A\,x_t &= B\,x_{t-1} + I\epsilon_t \\
x_t &= A^{-1}B\,x_{t-1} + A^{-1}I\epsilon_t \\
x_t &= C\,x_{t-1} + D\epsilon_t
\end{aligned}
$$

where $C = A^{-1}B$ and $D = A^{-1}I$. For stable solution, the eigenvalues of matrix C must be stable, that is $|\theta_i| < 1$ (θ_i denotes eigenvalues).

If the system is stable, we can simulate and study its behaviour by impulse responses (reaction of variables to different types of shocks). We assume that all variables are on their steady state values (the deviation is equal to zero). If the shock hits the economy the variables depart from the steady state path and we can examine their behavior over the cycle. We are usually interested in direction and magnitude of the deviation and length of the adjustment process.