Okun's law

Okun's law was empirical relationship between deviation of GDP from trend and change in the unemployment rate. It can be written in log-linearised form as

$$u_t - u_{t-1} = -\beta(y_t - \bar{y}_t) = -\beta\hat{y}_t$$

where β was between 1/4 and 1/2 (dependent on observed period). Nowadays, Okun's law is understood as stochastic relationship between gap of GDP and deviations of unemployment from its trend or from natural rate of unemployment (often called NAIRU – Non-Accelerating-Inflation-Rate-of-Unemployment). For convenience we will write it in reversed form:

$$\hat{y}_t = \gamma(u_t - \bar{u}_t) + \epsilon_t,$$

where $\hat{y}_t$ is gap of GDP, $u_t - \bar{u}_t$ is deviation of unemployment rate from trend, ϵ_t is residual and γ is (negative) descriptive parameter (we do not know its economic interpretation, yet).

This is model in reduced form. We can estimate it on real data, but the main purpose of macroeconomic analysis is to find structural (behavioral) foundations.

Derivation

We suppose that output is produced by (Cobb-Douglas) production function with two factors (capital and labor) and neutral technology progress.

$$Y_t = F(A_t, K_t, L_t) = A_t K_t^{1-\alpha} L_t^{\alpha}$$

where Y_t is output, K_t is capital, L_t is labor, A_t represents technology progress and α is labor intensity of production, $\alpha \in (0, 1)$.

We suppose that cyclical fluctuations are caused by only one production factor – labor. Capital K_t and technology progress A_t are exogenous and thus fixed at their steady state level. Output and labor can be decomposed into trend values and percentage deviations:

$$Y_t = \overline{Y}_t(1 + \hat{y}_t), \qquad L_t = \overline{L}_t(1 + \hat{l}_t)$$

The production function can be written as:

$$\overline{Y}_t(1 + \hat{y}_t) = \overline{A}_t \overline{K}_t^{1-\alpha} \overline{L}_t^{\alpha}(1 + \hat{l}_t)^{\alpha}$$

If for the trend (steady state) values holds

$$\overline{Y}_t = \overline{A}_t \overline{K}_t^{1-\alpha} \overline{L}_t^{\alpha}$$

we can get

$$1 + \hat{y}_t = (1 + \hat{l}_t)^{\alpha}$$

After taking logarithm and using approximation the result is

$$\hat{y}_t = \alpha\, \hat{l}_t. \tag{1}$$

Now we need to find approximation of relationship between gap of employed labor $\hat{l}_t$ and gap of unemployment rate $\hat{u}_t = u_t - \bar{u}_t$. We can use the definition:

$$\hat{l}_t = \frac{L_t - \overline{L}_t}{\overline{L}_t}$$

If we denote $F_t = L_t + U_t$ as the total labor force (amount of employed people plus the amount of unemployed people) and if we assume that U_t is only small part of the total labor force F_t we can get

$$\hat{l}_t = \frac{L_t - \overline{L}_t}{\overline{L}_t} \approx \frac{L_t - \overline{L}_t}{F_t} = \frac{(L_t - F_t) - (\overline{L}_t - F_t)}{F_t} = \frac{(-U_t) - (-\overline{U}_t)}{F_t} = -(u_t - \bar{u}_t) = -\hat{u}_t$$

where $u = U_t/F_t$ is the definition of unemployment rate. After inserting into equation (1) the result is

$$\hat{y}_t = -\alpha\, \hat{u}_t \tag{2}$$

In this relationship, parameter α has economic interpretation, it is labor intensity of production and it should lie in interval $(0, 1)$.

Exercises

1. Estimate regression model

$$\hat{y}_t = -\alpha \hat{u}_t + \epsilon_t,$$

where $\hat{y}_t$ is gap of GDP and $\hat{u}_t$ is gap of unemployment rate (use HP filter for decomposition).

2. Estimate model

$$\hat{y}_t = -\alpha(u_t - \bar{u}) + \epsilon_t,$$

where expression in brackets is deviation of unemployment rate from natural rate of unemployment $\bar{u}$. Suppose that natural rate is constant in time.

3. Estimate reversed model

$$u_t = \bar{u} - \frac{1}{\alpha}\hat{y}_t + \omega_t,$$

where $\bar{u}$ is again constant value of natural rate of unemployment.

4. Modify previous model by introducing rigidities to labor market

$$u_t = w\, u_{t-1} + (1-w)[\bar{u} - \frac{1}{\alpha}\hat{y}_t] + \omega_t,$$

where actual unemployment rate u_t depends also on lagged unemployment rate u_{t-1}. w is weight to lagged value and expresses rigidities on labor market. $(1-w)$ is weight to the previous model.