Stationarity

is a common assumption in many time series techniques. Time series observed in the practise are sometimes non-stationary. In this case, they should be transformed to some stationary time series, if possible, and then be analysed. Two types of stationarity exists: strong (or strict) and weak stationarity. Weak stationarity is sufficient for our purposes.

A weak stationary process has the property that the *mean, variance and autocovariance* structure do not change over time.

In mathematical terms:

$$E(X_t) = \mu \quad \text{for all } t$$

$$E(X_t^2) = \sigma^2 \quad \text{for all } t$$

$$\text{cov}(X_t, X_k) = \text{cov}(X_{t+s}, X_{k+s}) \quad \text{for all} \quad t, k, s$$

In other words, we mean flat looking series, without trend, with constant variance over time and with no periodic fluctuations (seasonality) or autocorrelation. There are several formal tests of stationarity;[1] quite popular is Augmented Dickey-Fuller test.

Non-stationary data: example

Remember data series of consumption in the USA and government spendings in Greece. These time series are non-stationary. If we calculate coefficient of correlation the result is 0.96. This result indicates very strong relationship between these data. But this result is evidently misleading. The reason is non-stationarity of these time series.

Correlation coefficient ρ shows how closely two variables move together; mathematical formula (for theoretic value) is

$$\text{cov}(X, Y) = E[(X - E(X))(Y - E(Y))]$$

$$\text{var}(X) = \sigma_X^2 = E[X - E(X)]^2$$

$$\rho(X, Y) = \frac{\text{cov}(X, Y)}{\sigma_X \sigma_Y}$$

The range of correlation coefficient can be from 1 (positive correlation) to -1 (negative correlation); a value of zero indicates the absence of correlation. Correlation coefficient (in a simplyfied way): we calculate mean value of the time series then compute deviations of every element from the mean and make comparison. If there are two time

[1] Test for presence of unit root.

series, that systematically growth, there are large negative deviations from mean value at the beginning of time series and large positive deviations at the end of both time series, which implies high value of correlation. There are several ways to cope with this problem.

Stationarization

- differentiation of time series
- filtration – estimation of trend of time series and working with the gap (deviation from trend).

The choice depends on economic theory. Is is reasonable to calculate logarithm of time series and then:

- For time series of variables that are fundamently determined i.e. **real** variables (real GDP, real wages, real money balances, real exchange rates) use **filtration** to extract trend and then work with the gap.
- For **nominal** variables (e.g. price indices, nominal exchange rates) use **differentiation** and then work with percentage changes.

This type of analysis (analysis with stationary data) is good for cyclical (not long-run) behavior of time series. Sometimes we want to examine long-run relationship between economic variables. This type of analysis is more complicated and comes under cointegration analysis (error correction modelling). We will briefly deal with this method later.

Differentiation

Differentiation of logarithm of time series produces aproximation of the growth rate.

Last lesson we computed inflation rate (growth rate of price level) as the first difference of logarithm of price level:

$$\pi_t = \Delta \log P_t = \log P_t - \log P_{t-1}.$$

One problem connected with differentiation is what length of lag to choose. From quarterly data we can compute year-on-year changes or quarter-on-quarter changes.

Year-on-year change is often used to eliminate the problem with seasonality; such time series is also more transparent and not so volatile. However, this way of differentiation introduce artficial autocorrelation into time series and the results of the analysis can be distored. Furthermore, from the point of view of economic theory there is no reason to compute other types of changes than quarter-on-quarter (or month-on-month) i.e. differences to the previous period.

Let's assume random walk process:

$$x_t = x_{t-1} + \epsilon_t$$

where ϵ_t is white noise, $\epsilon_t \sim WN(0, \sigma_\epsilon^2)$, stationary process with zero mean, constant variance σ_ϵ^2 and no autocorrelation. First difference (to previous period) of this process is

$$\Delta x_t = x_t - x_{t-1} = \epsilon_t$$

which implies, by definition, no autocorrelation

$$\mathrm{cov}(\Delta x_t, \Delta x_{t-1}) = \mathrm{cov}(\epsilon_t, \epsilon_{t-1}) = 0$$

Computing difference over four periods (like year-on-year changes) results in

$$\Delta_4 x_t = x_t - x_{t-4} = \epsilon_t + \epsilon_{t-1} + \epsilon_{t-2} + \epsilon_{t-3}$$

and the correlation is then

$$\mathrm{cov}(\Delta_4 x_t, \Delta_4 x_{t-1}) = \mathrm{cov}(\epsilon_t + \epsilon_{t-1} + \epsilon_{t-2} + \epsilon_{t-3}, \epsilon_{t-1} + \epsilon_{t-2} + \epsilon_{t-3} + \epsilon_{t-4}) = 3\sigma_\epsilon^2$$

Filtration

Filtration is a decomposition of time series into trend and cyclical component. We will use the Hodrick-Prescott filter[2] which is a smoothing method, widely used among macroeconomists, to obtain a smooth estimate of the long-term trend component of a series.

It is reasonable to use HP filter on logarithm of time series, so that the first difference has meaning of the growth rate and the gap (cyclical component) is then expressed as percentage deviation from trend.

Assumptions of HP filter:

$$x_t = \bar{x}_t + \hat{x}_t$$

[2] Hodrick, R. J., Prescott, E. C., Postwar U.S. Business Cycles: An Empirical Investigation *Journal of Money, Credit and Banking*, no. 11, p. 1 - 16, 1997.

$$\Delta\bar{x}_t = \Delta\bar{x}_{t-1} + \omega_t$$

$$\hat{x}_t = \epsilon_t$$

where x_t is (log of) original time series, $\bar{x}_t$ is trend component, $\hat{x}_t$ is cyclical component (gap), ω_t is a shock (disturbance) in the growth rate of trend component, ϵ_t is a cyclical shock. We assume that both shocks are white noises. Formally written: $\epsilon_t \sim WN(0, \sigma_\epsilon^2)$, $\omega_t \sim WN(0, \sigma_\omega^2)$.

The only optional parameter, that this filter uses, is the ratio of variance of cyclical shock and variance of trend shock.

$$\lambda = \frac{\sigma_\epsilon^2}{\sigma_\omega^2}.$$

The larger is this parameter (λ), the smoother is the trend. When the parameter is close to infinity, the trend is linear. On the other hand, when λ is small (close to 0) the trend more follows original time series. The authors of the filter, Hodrick and Prescott, recommend to use these values of λ: for annual data 100, for quarterly data 1600 and for monthly data 14400.

Decomposition of GDP

Original value of real GDP is Y_t, trend component is $\bar{Y}_t$, gap (cyclical component) $\hat{y}_t$ is expressed as percentage deviation from trend (0,10 means 10 %).

$$Y_t = \bar{Y}_t(1 + \hat{y}_t)$$

Taking logarithm of both sides

$$\log Y_t = \log \bar{Y}_t + \log(1 + \hat{y}_t)$$

using logarithmic approximation[3] and denoting logarithm of original values with small letters produces

$$y_t = \bar{y}_t + \hat{y}_t$$

which is exactly the decompositions of variable for HP filter.

[3] $\log(1 + x) \approx x$ for small values of x (to 0.15)

Demand and supply shocks

Basic categories of shocks

Demand shocks affect demand side of the economy (e.g. increase of government expenditures). In AS-AD framework it corresponds to shifts of AD schedule. In time representation, the lines representing inflation rate and the gap of GDP are going in the *same* direction.

Supply shocks affect supply side of the economy (e.g. increase in costs of firms – oil shocks). Shifts of AS schedule in AS-AD model or the lines representing inflation rate and the gap of GDP are going in the *opposite* direction.

Both demand and supply shocks can be positive or negative. Some of the shocks can affect both sides of the economy (e.g. change of tax system) and thus distinguishing into demand and supply shocks is not appropriate. More detailed analysis of the shocks is usually based on model structure (the more complex model, the more number of shocks). List of potential shocks can include e.g. consumption preference shock, labor supply shock, wage markup or price markup shock, risk premium shock, productivity shock, government spending shocks, inflation target shock, shock in terms of trade ...

Can the shocks be sources of **inflation**? As the initial cause, yes. However, inflation is monetary phenomenon and is connected with excessive supply of money. Shock can cause increase (or decrease) of inflation, but only temporarily. Emission of money (and central bank) is responsible for inflation in the long-run.

Deflation is sustained decrease in the price level (negative inflation rate).
Disinflation means reduction of inflation rate (usually by force of monetary policy).