Stylized facts

could be defined as **empirical regularities** (or some characteristic features that occur in the data). However, they do not need to be rigorously exact *everywhere* and *all the time*. They are useful because they help us to test implications of a theory or to choose between alternative theories. We can compare conclusions of a theory with stylized facts (empirical behaviour of economic variables) and judge usefulness of that theory.

Business cycles

Business cycle can be defined as the deviations of *aggregate real output* from trend. We can identify several phases of a typical cycle. Periods of expansions are called **boom**, periods of contraction are **recession**, the turning points are **peak** and **trough**.

Using Hodrick-Prescott filter we can estimate the trend of the GDP and compute deviation from the trend – the gap (expressed as percentage deviation). The gap represents cycle of real output.

The behaviour of GDP is used to identify business cycles. However, we are also interested in other macroeconomic variables and their behaviour over the cycle. We can apply the same procedure (filtration) also on other macroeconomic variables and make comparison with behaviour of GDP.

Cyclical behaviour

Comovement

The economic variable is said to be

- **procyclical** if its deviations from trend are *positively* correlated with the deviations from trend in the real GDP
- **countercyclical** if its deviations from trend are *negatively* correlated with the deviations from trend in the real GDP
- **acyclical** if it is neither procyclical nor countercyclical.

How to identify the comovement of these varaibles with real GDP? First, we can look at the plot of two time series:

- x is high when y is high: there is positive correlation between variables x and y.
- x is low when y is high: there is negative correlation between variables x and y.

Second, we can compute *correlation coefficient* that measure the degree of correlation. If the correlation coefficient is 1, then x and y are perfectly **positively correlated**. If the correlation coefficient is -1, then x and y are perfectly **negatively correlated**. If the correlation coefficient is 0, then x and y are **uncorrelated**. We can also distinguish if the variable is *strongly* or *weakly* correlated, e.g. for $|\rho| < 0.5$ is weakly correlated, and for $|\rho| > 0.5$ is strongly correlated. However, this distinguishing is rather subjective.

An important characteristic of comovement is the leading and lagging relationships that exist in macroeconomic data. If a macroeconomic variable tends to aid in predicting the future path of real GDP, we say it is a **leading variable**. If real GDP helps to predict the future path of a particular macroeconomic variable, then that variable is said to be a **lagging variable**. A **coincident variable** is one which neither leads nor lags real GDP.

The identification is possible by looking at the plot of time series but precise measure provides **cross correlation coefficient**. The cross correlation coefficient that we use is constructed as follows:

$$\text{corr} = \begin{cases} \rho(x_t, y_{t+\bar{k}}) & \\ \vdots & y \text{ lags} \\ \rho(x_t, y_{t+1}) & \\ \rho(x_t, y_t) & \text{coincide} \\ \rho(x_t, y_{t-1}) & \\ \vdots & y \text{ leads} \\ \rho(x_t, y_{t-\bar{k}}) & \end{cases}$$

where $\bar{k}$ is maximum of the phase shift (arbitrary set, usually $\bar{k} = 5$).

For the relationship between x_t and y_{t+k},[1] the largest *absolute* value of correlation coefficient in period k indicates that the variable y is

- lagging the cycle by k periods when $k > 0$,

[1]The variable x_t represents output gap, whereas y_t is gap of the variable of interest.

- leading the cycle by k periods when $k < 0$,
- coincide the cycle when $k = 0$

Variability

Finally, there are key regularities in terms of the variability of economic variables over the business cycle. A measure of cyclical variability is **standard deviation**[2] (of the gap of the variable).

Testing of statistical significance

The hypothesis that two variables x_t and y_t are independent (uncorrelated) is equivalent to the hypothesis $\rho(x_t, y_t) = 0$. The test of statistical significance of correlation coefficient is based on Student test. The procedure is as follows: We compute correlation cofficient, then compute value of t-statistic using following formula

$$t_{stat} = \frac{\rho}{\sqrt{1-\rho^2}}\sqrt{n-2}$$

where n is number of observations. Then we compare t-statistic with critical value of Students t-distribution $t_{\alpha/2}(\nu)$ where α is level of significance (usually $\alpha = 0.05$) and ν is number of degrees of freedom ($\nu = n - 2$). If

$$|t_{stat}| > t_{\alpha/2}(\nu)$$

we reject the hypothesis of independence and accept the hypothesis that ρ is significantly different from zero. In other words, we conclude that the correlation coefficient is statisticaly significant and the variables are correlated.[3]

[2]Standard deviation $\sigma_X$is the square root of the variance $\sigma_X^2 = E[X - E(X)]^2$

[3]In the business cycle terminology, we want to distinguish whether the variable is procyclical/countercyclical (significant corr. coef.) or acyclical (not significant corr. coef.).