Interest parity condition

Nominal exchange rate is relative price of two nominal assets. As any other price of asset, it is determined by expected rate of return from that asset. Suppose two currencies, home and foreign, and two identical instruments on financial market. These assets are perfect substitutes (later we will relax this assumption), one is denominated in home currency the other is denominated in foreign currency. The return on the asset is payable next period and is expressed by nominal interest rate.[1] Investors at financial market compare expected rates of return from these assets and invest their money into asset that brings higher expected return.

Example

If you invest one unit of home currency in time t into home asset, you will earn $(1+i_t)$ in time $t+1$. If you want to invest one unit of home currency into foreign asset you have to convert it at nominal exchange rate S_t.[2] After conversion of the money (one unit of home currency is $1/S_t$ of foreign currency), investment into foreign asset brings you in time $t+1$ return of

$$\frac{(1+i_t^*)}{S_t}$$

But this return is expressed in foreign currency (e.g. dollars). To compare your investment, you must compare the returns in the same currency. You must convert the return of investment back into domestic currency, at exchange rate in time $t+1$.

$$\frac{(1+i_t^*)S_{t+1}}{S_t}$$

However the future exchange rate is not known in time t, it can be only anticipated (we don't assume existence of market with forwards or some other instruments that ensure you against the risk connected with the change of exchange rate). In the equation above the expected exchange rate $S_{t+1|t}^e$ appears.

   Investors compare expected rates of return from these two financial instruments. If there is any difference between the returns, financial flow toward more attractive asset (increased demand) would cause appreciation of that currency and elimination of the

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[1] Foreign interest rate is denoted with asterisk.

[2] Exchange rate is quoted as home currency units per one unit of foreign currency e.g. CZK/USD. Increase of exchange rate means depreciation.

gap between the rate of returns. The only possible outcome is equality of the expected returns.

$$(1 + i_t) = (1 + i_t^*)\frac{S_{t+1|t}^e}{S_t}$$

This equation is called **uncovered interest parity condition** (UIP) and it is so called *no arbitrage condition*. It generally means that it is not possible to earn some extra profit by doing business with the same good or asset in the same time in two different markets. If any such possibility appears, effective market would exploit this opportunity and it would immediately disappear.

For transparency we can work with approximation of UIP, after taking logarithms:

$$i_t - i_t^* = s_{t+1|t}^e - s_t \tag{1}$$

where $s_t = \log S_t$. The left hand side of this equation is differential of interest rates, the right hand side is expected future depreciation of exchange rate (of home currency).

However, in reality, we can observe significant and systematic deviations from validity of UIP. They can be approximately estimated by assuming that mean value of difference between expectations and ex-post value of exchange rate is zero.

$$E(s_{t+1|t}^e - s_{t+1}) = 0$$

It means that the market expectations are not mistaken on average. After inserting realized value of s_{t+1} instead of s_{t+1}^e in the equation (1) we can analyse the disparity – when the UIP is not valid.

Application

The equation of disparity for three month interbank interest rate in the Czech Republic and Germany (PRIBOR and FIBOR) and exchange rate CZK/DEM is[3]

$$\Psi_t = i_t^{CZ} - i_t^{GER} - 4\left(s_{t+3} - s_t\right)$$

The most common interpretation of the disparity is existence of risk premium. **Risk premium** is consequence of imperfect substitution between home and foreign assets. The investors can consider investment into home deposits more risky and thus they want compensation in the form of higher expected rate of return. The higher risk premium, the higher disparity in UIP.

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[3]The depreciation of exchange rate is multiplied by 4, because we convert change over three months into annual expression, because the interest rates are quoted per annum.

The risk premium is behaviorally connected with:

- volatility of exchange rate – higher volatility of exchange rate means lower ability to predict it and thus less certainty about the return on domestic assets

- volume of the foreign debt of the country – higher debt (relative to some measure of performance, e.g. GDP) means higher risk that economic agents will not be able to serve the debt sometimes in the future.