Agent-based Computational Model of Democratic Choice of Redistribution under Almost Perfect Human Mobility

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Abstract. This paper shows by means of an agent-based simulation that people's ability to relocate from one region to another creates no tax competition among the regions if the taxes and transfers are set democratically. We can observe three types of behavior: 1) if the cost of relocation is high, no one relocates and the original level of redistributions is sustained; 2) if the cost is low, the system has no steady state—the more productive people run away from redistribution and the less productive ones pursue them forever; 3) if the cost is medium, usually only the more productive people relocate—the tax rates usually fall and converge (but not always). If they do, it is because the populations are more homogenous after the relocation, not because of tax competitions—there is none.

Keywords: redistribution, democracy, human mobility

JEL classification: H23, H26, H30, C63

1 Introduction

The goal of this paper is to explore whether human mobility restricts involuntary redistribution via taxation and transfers when the taxes and transfers are set up democratically by the majority vote. A simple intuition is that it does because there would be tax competition among regions. A region with a high level of redistribution would lose its relatively more productive inhabitants (net payers) and attract relatively less productive inhabitants (net receivers). This would force the region to lower its level of redistribution. However, such intuition is based on an implicit assumption that regions are entities pursuing some goal (as if governed by dictators). If the level of redistribution is set up democratically by the majority vote, a more complex dynamics arises. When a person relocates from her region, she changes 1) the amount of taxes paid and transfers demanded in her old and new home regions and 2) the voting majority in each of these regions. This could change the level of redistribution both in her old and new regions, which can make other persons in these regions uncomfortable, and thus provoke the next wave of relocation. This process might be repeated many times until the steady state is reached—if there is any kind of a steady state at all.

This complex problem involving heterogenous locally interacting agents could hardly be modeled with standard modeling tools. Therefore I have constructed a simple agent-based computational model that allows us to analyze it by means of a computer simulation (for introduction to agent-based computational economics see [10]). The model shows that even an extremely simple model specification leads to an interesting complex (possibly non-equilibrium) dynamics with phase transitions that contradicts the above mentioned intuition about the tax competition.

We can observe three phases: 1) if the cost of relocation is high, no one relocates and the original level of redistributions is sustained; 2) if the cost is low, the system does not settle down—the more productive people run away from redistribution and the less productive ones pursue them; 3) if the cost is medium, usually only the more productive people relocate—the tax rates usually fall and converge (but not always). If they do, it is because the populations of the regions are more homogenous after the relocations and vote for lower taxes. It is not because of tax competition—there is none since there is

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no central agent in a region able to set the tax rates independently of the will of the region's present inhabitants. Thus the intuition that regions play a strategic game is misleading.

There is a vast literature on tax competition: [12] provides a very good review, [7] provides a broader picture. Most of the literature deals with the question of efficiency of the tax competition. One strand of the literature, originating from Tiebout [11], claims that the tax competition is efficiency enhancing; the other one, originating from Oates [6] claims the opposite. The reason is that the tax competition leads to inefficient resource allocation and "race to the bottom" under most conditions: the competitive governments provide less than efficient amount of public goods, social security, environmental protection etc. For the implications for the European Union see [9]. However, recently there have appeared many papers showing that if there is another inefficiency (non-benevolent government, imperfect competition etc.), the tax competition could in fact be efficiency enhancing, see e.g. [1] or a good exposition in [8, pp. 325–339]. In any case, there is a little doubt (theoretic or empirical) that the tax competition exists. For empirical estimates for Europe see e.g. [4].

Most of the literature models the tax competition as a strategic conflict among (local) governments. The government is usually supposed to care about the welfare of its public modeled as a representative household (for non-benevolent government see e.g. [1] or [8]). However, some papers (e.g. [2] or [3]) model the tax competition as a problem of democratic choice. In Epple and Romer's paper [2], a continuum of heterogenous households scatters among a finite number of regions based on the regions' tax / transfers policy which is set up democratically by the regions' current inhabitants. Though the paper motivates the tax competition differently from this paper, some of its conclusions are similar to mine: the voters segregates in accordance with their incomes and there is no equilibrium under some conditions. Hindriks' paper [3], though different in many respects, has two results similar to mine: 1) in symmetric equilibrium, "the rich are mobile and the poor chase them" [3, p. 109] and 2) in the asymmetric equilibrium, the rich and the poor inhabitants are segregated, though imperfectly.

The original contribution of this paper is as follows: 1) there need not to exist any tax competition (as strategic game) to explain the data, 2) the existence of the equilibrium and its properties depend on the cost of relocation (compare to Tiebout's claim that "the cost of moving from community to community should be recognized" [11, pp. 421–422]), 3) it shows what might be happening in the non-equilibrium cases, and 4) it provides an elementary agent-based model of tax competition which could be enhanced along several lines. To my knowledge, there was no agent-based model of the tax competition so far.

2 Description of the Model

The model world consists of regions and inhabitants. Each inhabitant lives in one region, produces a homogenous product in her own business, and pays part of it to government as taxes. She consumes all her product plus the net transfers (i.e. the transfers she gets from the government minus the taxes she pays to the government). There is direct democracy in each region, such that the tax rate is determined by the majority vote of all the inhabitants of the region. Anyone can propose a tax increase or cut. The proposal is passed if more than one half of the inhabitants of the region benefit from the change (i.e. if they vote for it). The inhabitants can relocate if they believe they would be better off in another region (taking into account a cost of relocation). I assume that inhabitants of a region cannot raise arbitrary barriers to protect themselves either from inflow or immigrants, or to restrain their fellow citizens' emigration. There is no trade or credit among the inhabitants.

More formally, we assume there are K regions and N inhabitants. There are $N_{t,j}$ inhabitants in region j at time t ($j = 1, \ldots, K$). Let us denote $A_{t,j}$ the set of indices of all inhabitants of region j in time t. All inhabitants of region j pay a flat-rate tax. The proceedings are redistributed among the region j's inhabitants as lump-sum transfers. The region j's budget constraint is then

$$N_{t,j} \cdot v_{t,j} = \sum_{i \in A_{t,j}} \tau_{t,j} \cdot y_{t,i} \tag{1}$$

where $\tau_{t,j} \in \{0, 0.01, 0.02, \ldots, 0.99, 1\}$ is the tax rate in region j at time t, $v_{t,j}$ is the amount of transfers in region j at time t, and $y_{t,i}$ is the product of the region j's inhabitant i at time t. Region j's tax system thus can be described as a couple $(\tau_{t,j}, v_{t,j})$. (Budget deficits or surpluses are not allowed.)

Every inhabitant i produces a homogenous product $y_{t,i}$ at time t. Her production function f is

$$y_{t,i} = f(l_{t,i}) = a_i \cdot l_{t,i} \tag{2}$$

where $l_{t,i}$ is her labor effort at time t (labor is the only factor of production) and a_i is her productivity. The productivity is invariant in time, but differs among the inhabitants; it is the only source of heterogeneity among the inhabitants.

Each inhabitant i maximizes her utility function

$$u(c_{t,i}, l_{t,i}) = \sqrt{c_{t,i}} + \sqrt{1 - l_{t,i}} \tag{3}$$

subject to constraints

$$c_{t,i} = (1 - \tau_{t,\nu_{t,i}}) \cdot y_{t,i} + v_{t,\nu_{t,i}} = (1 - \tau_{t,\nu_{t,i}}) \cdot a_i \cdot l_{t,i} + v_{t,\nu_{t,i}}, \quad l_{t,i} \in [0, 1] \tag{4}$$

where $(1 - l_{t,i})$ is her leisure time at time t, $c_{t,i}$ is her consumption at time t, and $\nu_{t,i}$ is an index function that returns the index of the region where she is presently living.

The inhabitant i's optimal labor effort $l^*_{t,i}(j)$ in the region j would be

$$l^*_{t,i}(j) = \frac{(1 - \tau_{t,j}) \cdot a_i}{1 + (1 - \tau_{t,j}) \cdot a_i} - \frac{v_{t,j}}{(1 - \tau_{t,j}) \cdot a_i \cdot (1 + (1 - \tau_{t,j}) \cdot a_i)}. \tag{5}$$

The optimal labor effort $l^*_{t,i}(j)$ (and hence optimal level of production $y^*_{t,i}(j)$) decreases in both the tax rate $\tau_{t,j}$ and the transfers $v_{t,j}$.

Let us denote $u^*_i(\tau_{t,j}, v_{t,j}, \gamma)$ the maximal level of utility that the inhabitant i can have given the tax system $(\tau_{t,j}, v_{t,j})$ and a cost γ

$$u^*_i(\tau_{t,j}, v_{t,j}, \gamma) = u(c^*_{t,i}(j) - \gamma, l^*_{t,i}(j)) \tag{6}$$

where $l^*_{t,i}(j)$ is the inhabitant i's optimal labor effort and $c^*_{t,i}(j)$ is her optimal level of consumption given the tax system $(\tau_{t,j}, v_{t,j})$, i.e. $l^*_{t,i}(j)$ and $c^*_{t,i}(j)$ are the levels of labor effort and consumption maximizing (3) subject to (4). The parameter γ will be explained below.

In specified moments, the inhabitants in each region may change the tax rate by voting. The voting proceeds in steps. In each step, there is a random proposal to change the tax rate by $\delta \in \{-0.05, -0.04, \ldots, 0.04, 0.05\}$ (each value drawn with the same probability). Let us denote the proposed tax rate in region j at time t as $\tau^P_{t,j} = \tau_{t,j} + \delta$. The Ministry of Finance of region j then calculates the value of transfers $v^P_{t,j}$ such that the region j's budget constraint (1) holds. The proposal is passed if the majority of region j's inhabitants are better off with the new tax scheme, i.e. if for more than $N_{t,j}/2$ of the region's inhabitants hold that $u^*_i(\tau^P_{t,j}, v^P_{t,j}, 0) > u^*_i(\tau_{t,j}, v_{t,j}, 0))$. In such a case we set $(\tau_{t+1,j}, v_{t+1,j}) = (\tau^P_{t,j}, v^P_{t,j})$; otherwise we set $(\tau_{t+1,j}, v_{t+1,j}) = (\tau_{t,j}, v_{t,j})$. The voting can be repeated many times. As the voting is repeated, the tax system in the region converges to the one that would be chosen by the median voter.

In other specified moments, the inhabitants can relocate from their home region to another region. Each inhabitant i wants to move to a region where she would obtain the highest level of utility. There is a relocation cost γ paid in terms of consumption. Each inhabitant i acts like this: let us denote her present home region j. Further, let us denote k the region where is her $u^*_i(\tau_{t,k}, v_{t,k}, 0)$ maximal. Inhabitant i then relocates from region j to region k if $u^*_i(\tau_{t,k}, v_{t,k}, \gamma) > u^*_k(\tau_{t,j}, v_{t,j}, 0)$; otherwise she stays in region j. When inhabitant i relocates from region j to region k, the index i is removed from set $A_{t+1,j}$ and added to set $A_{t+1,k}$, the counter $N_{t+1,j}$ is decreased by 1, the $N_{t+1,k}$ is increased by 1, and the index function $\nu_{t+1,i}$ returns k instead of j.

Whenever any inhabitant relocates, the Ministries of Finance of all regions recalculate the level of transfers so that the regions' budget constraint (1) hold. If all inhabitants relocates from region j, i.e. $A_{t,j} = \emptyset$, the region j's tax system $(\tau_{t,j}, v_{t,j})$ is set to $(0, 0)$.

The inhabitants are *boundedly rational* in three respects: 1) When they relocate they assume they would not change the tax rate or the level of transfers neither in their former home region, nor in their new one. 2) When voting for the tax rate in their home region, they do no take into account the possibility that the new tax rate or the level of transfers can attract some foreigners, or repel some of current inhabitants, which both could make the newly passed pair $(\tau_{t,j}, v_{t,j})$ unsustainable. 3) When considering relocation, they expect to stay in the target region for a fixed amount of time T (possibly forever). Intuitively, they have to pay a cost Γ once and for all when they relocate. To compare this cost with their consumption (repeated every period), they have set a unit cost γ such that Γ is the present value of annuity γ, i.e. $\Gamma = \sum_{l=1}^{T} \beta^l \gamma$ where $\beta \in [0, 1]$ is a discount factor. Fully rational inhabitants should

establish the proper value of the unit cost γ from the model; for our inhabitants, γ is a constant (a parameter of the model).

Because the model is an agent-based computational model (see [10]), it is not solved, but simulated. I describe the simulation only for one particular set of parameters because the limited length of this paper does not allow a more general treatment. The simulation proceeds like this: In the beginning of each simulation, the model is initialized. We create $N = 1000$ inhabitants, assign each inhabitant i a random productivity a_i drawn from the continuous uniform distribution over the interval $(0, 1]$, and position her randomly in one of $K = 2$ regions (she is positioned in each region with the same probability). Then the tax rate $\tau_{0,j}$ is set to 0.5 in each region, and we let the inhabitants in all regions to adjust the tax system via voting, which is repeated 60 times. Each simulation consists of two steps: 1) the inhabitants vote for the change of the tax system (the voting is repeated 60 times) and 2) each inhabitant is allowed to relocate (if she wishes to). These two steps are repeated 500 times, and we observe the evolution of the system. The only treatment variable is the relocation cost γ. The model was simulated for $\gamma = 0, 0.001, 0.002, \ldots, 0.05$. It was run 100 times for each value of γ; random seeds 1, 2, ... 100 were used. I omitted 8 % of simulations where the initial tax rate did not converged in the initialization step. Only one measure of redistribution is used here: the tax rate τ.

The model was implemented in NetLogo 4.1 (see [5]); see the interactive web interface of the model at http://www.econ.muni.cz/~qasar/models.html. Data were processed and analyzed in Matlab and in Gretl.

3 Results of the Selected Simulations

The initial values of tax rates $\tau_{0,j}$ are determined by the empirical distribution of productivity parameters a_i of inhabitants in each of the two regions. The mean initial tax rate in the sample is 11.39 %, its median 11.5 %, minimum 3 %, maximum 21 %, and the standard deviation 3.55 %. The tax rate differed among the regions in 91 % cases; the level of transfers was different even if the tax rate was the same. The initial tax rates can be explained by a linear regression model including three explanatory variables: median of productivity, average productivity, and variance of productivity (for OLS estimates see table 1). The intuition is that the redistribution is set such as to make the median voter indifferent. The more productive is the median voter, the more she loses from redistribution because she is more discouraged from working, and that is why she opposes redistribution more strongly. On the other hand, the higher spread of productivities among the inhabitants (and the higher average productivity), the more can the relatively unproductive inhabitants benefit from taxing their more productive neighbors. However, this is of secondary importance—the major part of the explained data variance was explained by the median of productivity (about 99.6 % measured by Theil's coefficient β).

Within the simulation, we can observe how the inhabitants relocate into regions where they would be presently better off, and then vote for a tax change. Even though the relocation process is governed by a very simple rule on the individual level, it is quite complex. The net payers (those inhabitants who are paying more taxes then are receiving transfers) could target the region with lower taxes because they would pay less and be less discouraged from working, or they can target the region with *higher* transfers if they would become net receivers there. The net receivers (those inhabitants who are paying less taxes than are receiving transfers) could target a region with higher transfers to get more, or the regions with lower taxes where they can benefit from being less discouraged from working. The region with higher transfers can be the region with a higher tax rate, or with the same or even lower tax rate but more

	estimate	standard error	t-ratio	p-value
constant	20.7462	2.21757	9.355	0.0000
median productivity	−237.020	4.01710	−59.00	0.0000
average productivity	0.187887	0.00684830	27.44	0.0000
variance of productivity	189.967	14.8681	12.78	0.0000

$s = 0.44$, $R^2 = 0.98$, $\bar{R}^2 = 0.98$, $F(3, 88) = 1902,508$

Table 1: OLS estimates of the initial tax rates in region 1.

productive inhabitants.

The inhabitants actually relocate to their target region only if they step across the threshold given by the relocation cost γ. This is more likely the lower γ is, the more extreme the inhabitants' productivity is (the more different it is from the productivity of the median voter in the region), and the higher is the difference between tax rates and transfers among the regions. Moreover, the threshold is relatively lower for more productive inhabitants than for less productive inhabitants even if the differences of their productivities from the productivity of the median voter are the same.

After the inhabitants have relocated, they vote for a tax change. The voting has no strategic feature—inhabitants of a region simply vote for proposals they can benefit from. In particular, there is no strategic behavior (and hence no tax competition) of the regions as a whole, because the inhabitants of a region do not take the other regions' tax system into account when they are voting. The new tax rates and transfers after the relocation depend only on the new empirical distribution of productivities in a region. For instance, let us suppose that the last relocation was such that the most productive inhabitants from region 1 relocated to region 2. The impact on the tax rates in the two regions depends on relative strength of two effects: how much the relocation affected the productivity of median voter in each region and how it affected the variability of productivities (and average productivity) of the inhabitants in each region. In majority of cases, the tax rate in region 2 decreases; the tax rate in region 1 can increase, decrease or not change at all there.

The simulation produces three different types of outcomes that depend on the cost of relocation γ. The relative frequencies of these phases can be seen in Figure 1. Phase 1 is observed when γ is high relative to the initial difference in taxes and transfers among the regions. In such a case, there is no change in the system. No inhabitant relocates, therefore there is no reason to change the tax rates at all, and the level of redistributions given by the initial empirical distribution of productivity with the regions is sustained.

Phase 2 is observed when γ is very low relative to the initial difference in taxes and transfers between the regions. In this phase, the system does not settle down. Let us call the region which initially had the lower tax rate A, and the other region B. In the first step, the relatively more productive inhabitants move to region A, while the relatively less productive move to the region with higher transfers, which usually is region B. Consequently, the tax rate in region A decreases further, and in region B it often rises. The transfers can move in the opposite direction. The change in the tax systems provokes other inhabitants to relocate. The tax rate in region A declines for some time. However, the less and less productive inhabitants come to region A, and the productivity of the median voter declines while the spread of productivities rises there. Consequently, the tax rate starts to rise in region A at some moment. The opposite happens in region B. As the most productive inhabitants leave, the productivity of the median voter decreases, which for some time can increase the tax rate. However, the spread of productivities decreases there as well, which causes the tax rate to decrease at some point. At some moment, the situation reverses: region A which initially had lower taxes and attracted inhabitants has now higher taxes and loses inhabitants; for region B holds the opposite. (In the simulation described in this paper, this usually happens when

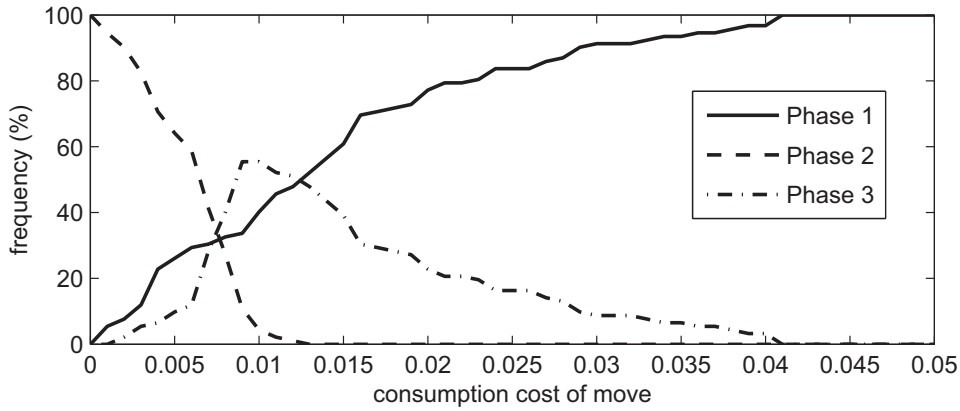

Figure 1: Relative frequencies of the phases observed in simulations depending on the cost of relocation γ.

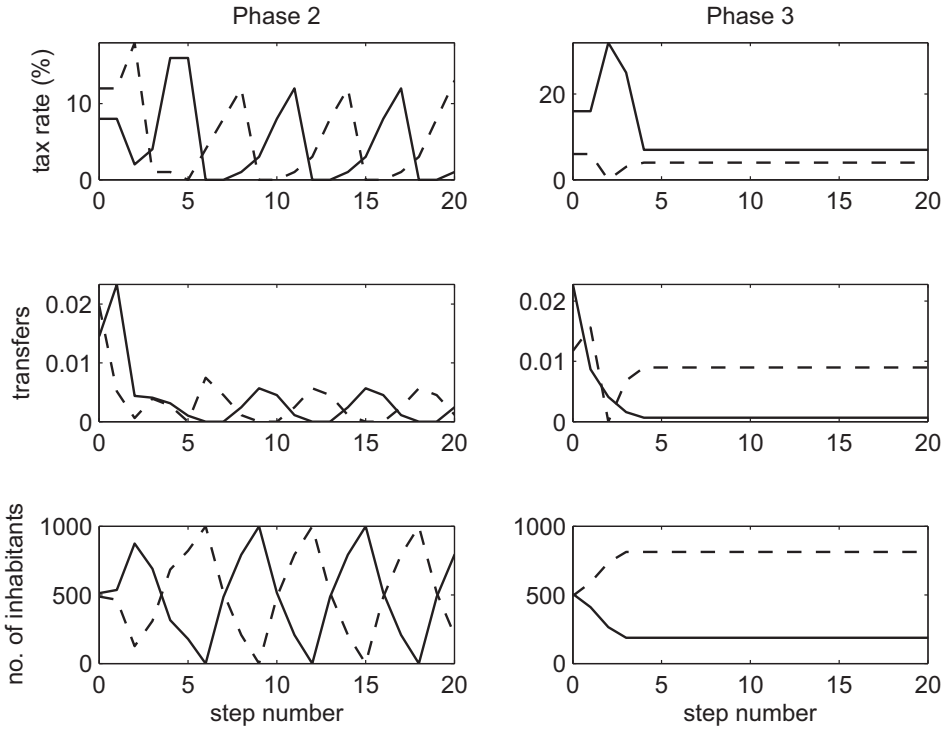

Figure 2: Typical evolution of the tax rate (top), transfers (middle), and number of inhabitants (bottom) in phase 2 (left) and phase 3 (right). The continuous line is used for region 1, the dashed line for region 2.

region B loses *all* of its inhabitants and its tax rate is set to zero. This is because the inhabitants relocate instantly. Simulations with some persistence in location show the same behavior of tax rates without the need to empty one region.) Thus regions A and B swap their positions, and a new cycle starts. A stable pattern of cycles appears: the more productive inhabitants run away from the forced redistribution and the less productive ones pursue them. Taxes and transfers in each region rise and then fall in cycles forever. For a typical evolution of the system see Figure 2, left panel.

Phase 3 is most interesting (and perhaps realistic). It is observed between the two extremes mentioned above. The evolution of this phase is similar to Phase 2 with two differences: 1) fewer inhabitants relocate because of the cost γ and the most productive inhabitants are most likely to relocate, and 2) the process of relocations stops when the differences between the regions' tax systems are two small that it does not pay off to any inhabitant to relocate (given the cost γ). Thus there is a steady state which is different from the initial setting. It may take some cycles to reach it, but usually the steady state is reached very soon with no cycles. In the steady state, the tax rates usually fall and converge (are lower and closer to each other than they were initially), but not always. If they do, it is because the regions' populations are more homogenous after the relocations. In 50 % of the simulations, the tax rates in both regions decreased; in 14 % of the simulations one decreased and one did not changed; in 17 % one tax rate increased and the other decreased; in 0.6 % no tax rate changed; in 16 % of the simualations one tax rate increased while the other did not changed; and in 1 % of the simulations the tax rates increased in both regions. If the tax rates were initially different, their difference decreased (i.e. they converged) in 61 % of the simualtions, it remained the same in 5.5 % of the simulations, and it increased in 33.1 %. If the tax rates were initially different, their order remained the same (i.e. the region with the initially lower tax rate had the steady state tax rate lower that the other region) in 83 % cases, their order switched in 11 % cases, and the rates were the same in 7 % cases. For a typical evolution of the system see Figure 2, right panel.

The overall conclusion is as follows: 1) There is no tax competition in its usual sense because there is nothing in the structure of the model that could produce it. 2) There are phase transitions; the phase depends on the relocation cost γ but it is not possible to sort the phases neatly just on the basis of γ

since the realized phase depends on the particular empirical distribution of productivities among regions. 3) Phase 1 is associated with high γ. There is no change in this case, and the tax rates are determined by the initial empirical distribution of the productivities among the regions. Phase 2 is associated with low γ. There is no steady state in this phase. The more productive inhabitants try to run from redistribution, and the less productive ones pursue them. Phase 3 is associated with the medium values of γ. In this phase, the more productive inhabitants relocate from the region which has initially higher tax rate to the other region where they usually lower taxes even more. The tax change in the other region is ambiguous. The tax rates decrease and converge in majority of cases in this phase (but not always). This is not an outcome of tax competition—it is caused by the fact that the populations of regions are more homogenous now.

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