Multi-Agent Simulation of Tiebout Model

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Abstract. In this paper, the Tiebout model (JPE, 1956) is explored by means of an agent-based computational simulation. The simulation suggests that the Tiebout's conjectures holds true only when the consumers are homogeneous in all respects other than their taste for public goods. If they are heterogeneous in other aspects (like productivity) too, some consumers try to parasitize on others and the others try to escape from the parasites. A system with small relocation cost does not reach an equilibrium. If the relocation cost is high enough, the system finds an equilibrium, but the equilibrium does not separate the consumers according to their taste for public goods. The aggregate quantity of public goods is lower than the optimal one and public goods and the tax burden are distributed among the inhabitants in an inefficient way. The system is preferred by a majority sufficient to establish it only under a rather restricting conditions.

Keywords: fiscal federalism, Tiebout's model, agent-based simulation

JEL classification: H210, H710, C690
AMS classification: 37M05, 68T42, 68U20, 91B12, 91B18

1 Introduction

In his seminal paper [4], Tiebout tried to solve the problem of the optimal provision of public goods in a world with heterogeneous consumers and imperfect information. The task was to make the consumers reveal their true "willingness to pay" for public goods. Tiebout's solution was ingenious: he proposed a decentralized system where public goods were provided by local governments. Each government would provide a different package of public goods and services and a different scheme to finance them. Each consumer would then patronize the region with the combination of the amounts of public goods and the tax structure she prefers most. Tiebout conjectured that this way the optimal level of public goods would have been provided (if there was a sufficient diversity among the regions) because each consumer would have had an incentive to reveal her true preferences. The essential condition for the system to work is that public goods are not truly non-rival, so the average cost of providing public goods is not ever-decreasing but is minimized by a certain quantity.

The problem with the Tiebout "model" is that it is only verbal. Its premises are not stated precisely, and its conclusions are not established properly. Moreover, there might be a serious flaw in the "model". In particular, it is not clear what Tiebout had in mind when he claimed that "[t]he consumer-voter may be viewed as picking that community which best satisfies his preference pattern for public goods" [4, p. 418]. A consumer may patronize a region for three reasons: 1) she likes the combination of public goods and the tax rate as such, i.e. she would patronize the region if it was populated by inhabitants with "tastes" for public goods and other characteristics identical to hers; or 2) she may patronize the region to "parasitize" on the contribution of the others, e.g. because the other inhabitants of the region are more productive than her, and thus pay higher taxes; she would not patronize the region if it was populated with inhabitants with the characteristics identical to hers; or 3) she may patronize the region to escape from "parasites" settled in a region which she would otherwise like better. It seems that Tiebout abstracted from similar interactions among the consumers. However, it has been shown in a different context (see e.g. [1]), that such interactions might change the relocation process and its outcomes considerably. The goal of this paper is to asses the validity of the Tiebout model for a world where consumers are heterogeneous both in their "tastes" for public goods and their productivity and where all above mentioned interactions among the consumers may occur. Since the problem involves heterogeneous, locally interacting agents, my approach is an agent-based computational simulation, see [3].

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2 Description of the Model

The model world consists of regions and inhabitants. Each inhabitant lives in one region, produces a homogeneous product in her own business, and pays part of it to the local government as taxes. She consumes what is left over from her production after the taxes are paid and public goods provided by the local government. There is no trade or credit among the inhabitants. The tax rate in each region is constant. The local government uses the the funds collected to provide a single kind of public goods produced with constant returns to scale. The inhabitants can relocate if they believe they would be better off in another region (taking into account a cost of relocation).

More formally, there are K regions and N inhabitants. There are $N_{t,j}$ inhabitants in region j at time t. Let us denote $A_{t,j}$ the set of indices of all inhabitants in region j in time t. All inhabitants of region j pay a flat-rate tax. The proceedings are used to provide public good in quantity $p_{t,j}$. The region j's budget constraint is then

$$N_{t,j} \cdot p_{t,j} = \sum_{i \in A_{t,j}} \tau_j \cdot y_{t,i}, \tag{1}$$

where τ_j is the tax rate in region j and $y_{t,i}$ is the product of the region j's inhabitant i at time t. Region j's tax system thus can be described as a couple $(\tau_j, p_{t,j})$. Budget deficits or surpluses are not allowed.

Every inhabitant i produces a homogeneous product $y_{t,i}$ at time t. Her production function f is

$$y_{t,i} = f(l_{t,i}) = a_i \cdot l_{t,i}, \tag{2}$$

where $l_{t,i}$ is her labor effort at time t (labor is the only factor of production) and a_i is her time-invariant productivity.

Each inhabitant i maximizes her utility function

$$u_i(c_{t,i}, p_{t,\nu_{t,i}}, l_{t,i}) = \sqrt{c_{t,i}} + \alpha_i \cdot \sqrt{p_{t,\nu_{t,i}}} + \sqrt{1 - l_{t,i}} \tag{3}$$

subject to constraints

$$c_{t,i} = (1 - \tau_{\nu_{t,i}}) \cdot y_{t,i} = (1 - \tau_{\nu_{t,i}}) \cdot a_i \cdot l_{t,i}, \quad l_{t,i} \in [0, 1], \tag{4}$$

where $\alpha_i \in (0, 1]$ is her time-invariant parameter of "taste" for public goods, $(1 - l_{t,i})$ is her leisure time at time t, $c_{t,i}$ is her consumption at time t, and $\nu_{t,i}$ is an index function that returns the index of the region where she is presently living. The inhabitants differ in two respects: in their productivity a_i and in their "taste" for public goods α_i (relative to their "taste" for private goods and leisure time).

The inhabitant i's optimal labor effort $l^*_{t,i}(j)$ in the region j is

$$l^*_{t,i}(j) = \frac{a_i(1 - \tau_j)}{1 + a_i(1 - \tau_j)}. \tag{5}$$

The optimal labor effort $l^*_{t,i}(j)$ (and hence optimal level of production $y^*_{t,i}(j)$) decreases in the tax rate τ_j. The amount of public goods $p_{t,j}$ provided in the region affects neither the optimal labor effort $l^*_{t,i}(j)$, nor the optimal level of production $y^*_{t,i}(j)$ because each inhabitant considers himself to be too small to affect the community's level of public goods.

Let us denote $u^*_i(\tau_j, p_{t,j}, \gamma)$ the maximal level of utility that the inhabitant i can have given the tax rate τ_j, the amount of public goods $p_{t,j}$ and a cost γ

$$u^*_i(\tau_j, p_{t,j}, \gamma) = u_i(c^*_{t,i}(j) - \gamma, p_{t,j}, l^*_{t,i}(j)), \tag{6}$$

where $l^*_{t,i}(j)$ is the inhabitant i's optimal labor effort and $c^*_{t,i}(j)$ is her optimal level of consumption given the tax rate τ_j, i.e. $l^*_{t,i}(j)$ and $c^*_{t,i}(j)$ are the levels of labor effort and consumption maximizing (3) subject to (4). The parameter γ will be explained below. (However, if $c^*_{t,i}(j) < \gamma$, then $u^*_i(\cdot) = -\infty$.)

In specified moments, the inhabitants can relocate from their home region to another one. Each inhabitant i wants to move to a region where she would obtain the highest level of utility. There is a relocation cost γ paid in terms of consumption. Each inhabitant i acts like this: let us denote j her present home region. Further, let us denote k the region where her utility $u^*_i(\tau_k, p_{t,k}, 0)$ is maximal. Inhabitant i then relocates from region j to region k if $u^*_i(\tau_k, p_{t,k}, \gamma) > u^*_k(\tau_j, p_{t,j}, 0)$; otherwise she stays in region j. (There is an exception when an inhabitant is considering a relocation into an empty region, where $A_{t,k} = \emptyset$, and hence $p_{t,k} = 0$. To allow the re-occupation of an empty region, I set its $p_{t,k}$ to the level it would have after the region k was patronized by the inhabitant i.) When inhabitant i relocates from region j to region k, the index i is removed from set $A_{t+1,j}$ and added to set $A_{t+1,k}$, the counter $N_{t+1,j}$ is decreased by 1, the $N_{t+1,k}$ is increased by 1, and the index function $\nu_{t+1,i}$ returns k instead of j. After each relocation step, all local governments recalculate the level of public goods provided in their regions so that the regions' budget constraint (1) hold.

The inhabitants are *boundedly rational* in two respects: 1) When they relocate they assume they would change the amount of public goods provided neither in their former home region, nor in their new one. Thus they ignore the possibility that their own relocation changes the amounts of public goods provided in the regions, which could provoke other inhabitants to relocate. They also ignore the possibility that other inhabitants can relocate at the same time too—there is no coordination among them. 2) When considering relocation, the inhabitants expect to stay in the target region for a fixed amount of time T (possibly forever). Intuitively, they have to pay a relocation cost Γ once and for all when they relocate. To compare this cost with their consumption (repeated every period), they have to set a unit cost γ such that Γ is the present value of annuity γ, i.e. $\Gamma = \sum_{l=1}^{T} \beta^l \gamma$ where $\beta \in [0, 1]$ is a discount factor. Fully rational inhabitants should establish the proper value of the unit cost γ from the model; but γ is a parameter of the model here.

The model is an agent-based computational model (see [3]), hence it is simulated, not solved. I describe the simulation only for one particular set of parameters because the limited length of this paper does not allow a more general treatment. The simulation proceeds like this: In the beginning of each simulation, the model is initialized. $K = 6$ regions are created, and assigned the tax rate τ_j equal to 0, 6, 12, 18, 24 and 30 % respectively. Then $N = 1000$ inhabitants are created. Each inhabitant i is assigned a random productivity a_i and a random "taste" for public goods α_i (a_i is drawn from the continuous uniform distribution over the interval $[0, 1]$ in one treatment and $[0.3, 1]$ in other treatment; α_i is drawn from the continuous uniform distribution over the interval $[0, 1]$). Each inhabitant i is positioned randomly in one of the regions (with the same probability in each one). When all inhabitants are created and located, each region j calculates the amount of public goods $p_{0,j}$ to be provided. Each simulation consists of 500 steps. In each step, each inhabitant is allowed to relocate (if she wishes to). When all inhabitants relocated, the amount of public goods provided in each region is recalculated. There are two treatment variables: the minimal productivity $a_{\min} = 0$ or 0.3 and the relocation cost $\gamma = 0, 0.002, 0.004, \ldots, 0.15$. The model was run 30 times for each value of the treatments with random seeds 1, 2, ..., 30. The model was implemented and simulated in NetLogo 4.1.2 (see [2]); data were processed and analyzed in R. The interactive web interface of the model is available at http://www.econ.muni.cz/~qasar/models.html.

3 Method of comparison to the Tiebout model

The comparison of the Tiebout model to the simulated model is not easy because the Tiebout model is stated only verbally, and hence imprecisely. For this reason, only qualitative comparison is possible. We can compare the models based on the Tiebout's four conjectures: 1) the strategic interactions among the inhabitants (parasitism and attempts to escape it) do considerably affect neither the relocation process, nor its outcomes, 2) the inhabitants patronize the regions which best satisfies their "tastes" for public goods, 3) public goods are provided in the optimal amount, and 4) the inhabitants prefer the Tiebout's world to the system with a uniform provision of public goods financed with a uniform tax rate set by a democratic vote. (The second and third claims are explicit, the first and fourth ones are implicit in the Tiebout's paper [4].) The first conjecture can be evaluated on two grounds: 1) the evolution of the simulated system can be observed directly, or 2) it can be claimed that the conjecture holds true if the other three conjectures hold true and vice versa.

The second conjecture apparently always holds true because the inhabitants always patronize their most preferred region. However, in the simulation model, each inhabitant takes into account the strategic considerations and the relocation cost when she chooses a region—the two aspects ignored by Tiebout. Thus to compare each inhabitant i's real location to her theoretic optimal one, I define inhabitant i's optimal region $\kappa(i)$ to be the region she would prefer most if all regions were populated with inhabitants identical to her (i.e. inhabitants with the same parameters a_i and α_i). Thus there would be no parasites and each inhabitant i would contribute precisely her share to public goods. That is, the most preferred region $\kappa(i)$ maximizes $u_i^*(\tau_j, p_{t,j}, 0)$ subject to constraint $p_{t,j} = \tau_j a_i l_{t,i}^*(j)$.

To test the Tiebout's third conjecture, the optimal level of public goods must be calculated. Tiebout did not define what amount of public goods is optimal when public goods are not completely non-rival. For the sake of comparison, I define the optimal aggregate level of public goods as the sum of public goods that would be provided to all inhabitants in all regions if each inhabitant i patronized her optimal region $\kappa(i)$. This quantity is compared to the real amount provided (the sum of public goods provided to all inhabitants in all regions where they are), and to the amount that would be provided to all inhabitants if there was a uniform tax rate and a uniform provision of public goods set democratically. The uniform democratic tax rate is calculated with a simple numeric algorithm.

In the outset, the uniform democratic tax rate is set to 100 %. If the number of inhabitant who prefer the tax rate to be lowered by 0.01 is at least one half of all inhabitants, the tax rate is lowered by 0.01. The procedure is repeated until less than one half of all inhabitants prefer the lower tax rate.

I also calculate how many inhabitants prefer the uniform democratic system to the Tiebout's hypothetic one, how many inhabitants prefer the uniform democratic system to the outcomes of the simulation model, and how many inhabitants prefer the Tiebout's hypothetic system to the outcomes of the simulation model. (Notice that these three preference relations need not to be transitive due to the Condorcet paradox.)

4 Results of the Selected Simulations

The results of the simulations are strikingly at odds with the Tiebout's conjectures. First, there is a phase transition in the model, see Figure 1. The realized phase depends on the relocation cost γ and the minimal productivity $a_{\min}$, though the phases cannot be classified neatly. The phases are three: 1) the model does not reach an equilibrium if the relocation cost is low—the inhabitants relocate forever; 2) no inhabitants relocate if the relocation cost is high; and 3) the equilibrium is reached after some inhabitants relocated if the relocation cost is intermediate. This fact shows that the strategic interactions among the inhabitants are important. If they were not, each inhabitant i would relocate to her optimal region $\kappa(i)$ and would stay there forever. But in fact, the less productive inhabitants pursue the more productive ones and the more productive inhabitants try to run away from them.

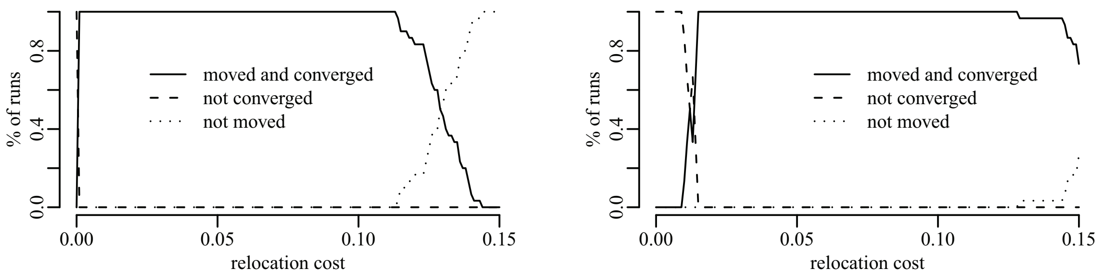

Figure 1: The phase transition in the model. The "not converged" denotes the share of runs where no equilibrium was reached. The "not moved" denotes the share of runs where no inhabitant relocated. The "moved and converged" denotes the share of runs where some inhabitants relocated and yet the equilibrium was reached. The left panel is for $a_{\min} = 0$, the right panel for $a_{\min} = 0.3$.

It is the relocation cost that stops the "hunting game" (if it is sufficiently high to do it at all). However, the inhabitants' equilibrium location is inefficient. For $a_{\min} = 0$ and low γ, most inhabitants relocate to low tax regions, usually to the region with the tax rate 6 %, see the left panel of Figure 2. It is because there are some extremely unproductive inhabitants in all regions which cannot relocate at all (their $c^*_{t,i}(\cdot) < \gamma$). The more productive inhabitants leave these regions to avoid parasitism, and can never invade these regions back because they are occupied by the unproductive inhabitants. The situation is similar but not identical for $a_{\min} = 0.3$. Most inhabitants end up in two or three regions in this case. One of them is the region with the tax rate 0 %, the other regions are picked randomly. That is why the mean occupation rate of all regions looks less diverse in the right panel of Figure 2 than in its left panel. When the relocation cost rises, fewer inhabitants relocate and more inhabitants stay in their original random position. Thus all occupation rates converge to its mean value 1/6. The same can be seen in Figure 3. When the relocation cost is low, most inhabitants are driven out to regions with lower than their most preferred tax rate (this effect is more pronounced when $a_{\min} = 0$). If the relocation cost is high, fewer inhabitants relocate and more stay in their original random region, but even in this case, more inhabitants end up in regions with lower then the most preferred tax rate than in regions with higher than the most preferred tax rate. It is because the more productive inhabitants can relocate more easily than the less productive ones, and they are biased in favor of the regions with lower taxes to avoid parasitism.

The amount of public goods that would by provided by a uniform democratic system is always higher than the hypothetic optimal amount, see Figure 4. It is because the inhabitants whose "taste" for public goods is low can "opt-out" of the system and because there is no parasitism in the Tiebout's hypothetic system too (contrary to the democratic system, see [1]). However, the really provided aggregate amount of public goods is even lower that the optimal one. It is especially low for $a_{\min} = 0$ and low γ because most inhabitants ends up in the regions with inefficiently low taxes in this case. The mean amount provided with $a_{\min} = 0.3$ and low γ is higher because the

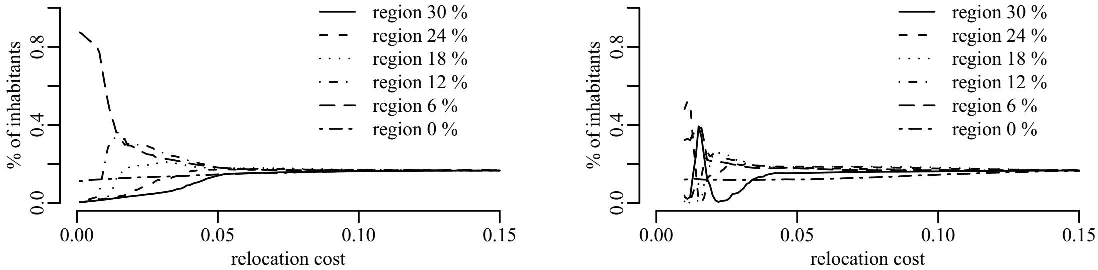

Figure 2: The equilibrium occupation rate of the regions in the runs where an equilibrium is reached. The left panel is for $a_{\min} = 0$, the right panel for $a_{\min} = 0.3$.

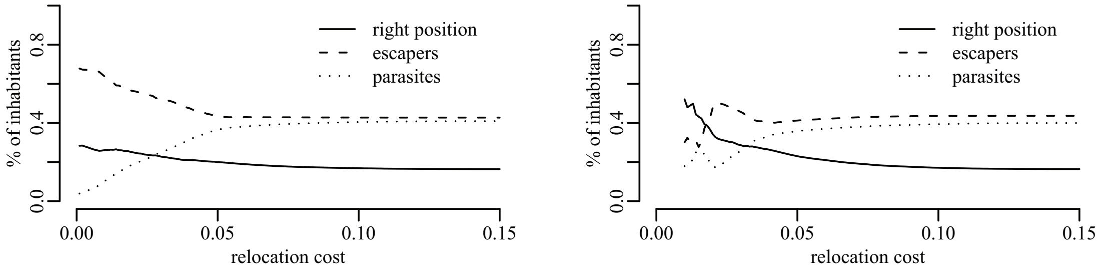

Figure 3: The mean frequency with which the inhabitants end up in their optimal region (denoted "right position"), in the region with a lower tax rate than in their optimal region (denoted "escapers"), or with a higher tax rate than in their optimal region (denoted "parasites"). Only the runs where an equilibrium is reached are considered. The left panel is for $a_{\min} = 0$, the right panel for $a_{\min} = 0.3$.

majority of the inhabitants ends up in random regions in this case, i.e. the tax rate paid by an average inhabitant is higher. As γ rises, the real aggregate amount of public goods converges to the optimal one. However, it is because most inhabitants stay in their original random location. Thus the aggregate amount of public goods is almost efficient but its distribution is random: inhabitants whose taste for public goods is high might pay a low tax rate and consume low quantity of public goods, and vice versa. In any case, the real aggregate amount of public goods is always lower than the optimal amount because the more productive inhabitants can relocate more easily than the less productive ones—and they tend to relocate to the regions with suboptimally low tax rates to avoid parasitism.

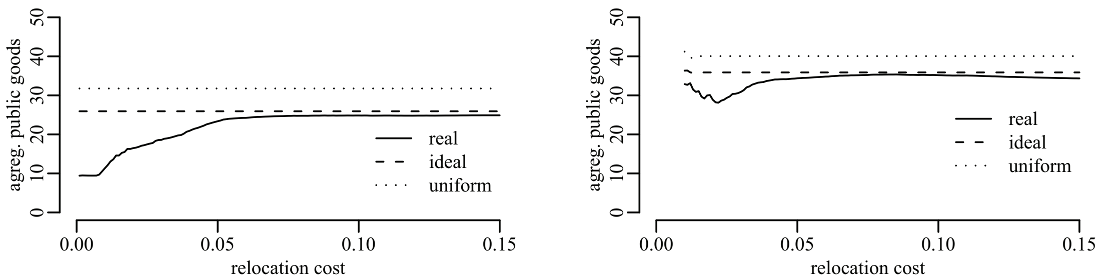

Figure 4: The mean aggregate amount of public goods provided. The "real" denotes the amount really provided in the model, the "ideal" denotes the Tiebout's optimal level of public goods, and the "uniform" denotes the amount that would be provided if all inhabitants paid the same tax rate established by a democratic vote. Only the runs where an equilibrium is reached are considered. The left panel is for $a_{\min} = 0$, the right panel for $a_{\min} = 0.3$.

The majority of inhabitants prefers the uniform democratic system to the Tiebout's hypothetic one and the hypothetic system to its real realization in the majority of runs, see Figure 5. For $a_{\min} = 0$, the majority also prefers the uniform democratic system to the real one in the majority of runs, hence the preferences are transitive. For $a_{\min} = 0.3$, the majority of inhabitants prefers the real system to the uniform in most runs when the relocation cost is low and yet the system reaches an equilibrium. When γ is higher, the majority prefers the uniform democratic system in most runs. However, the mean preference for the uniform democratic system to the Tiebout's ideal is mild, see Figure 6: roughly one half inhabitants prefer the uniform democratic system and the other half the Tiebout's hypothetic optimal system.

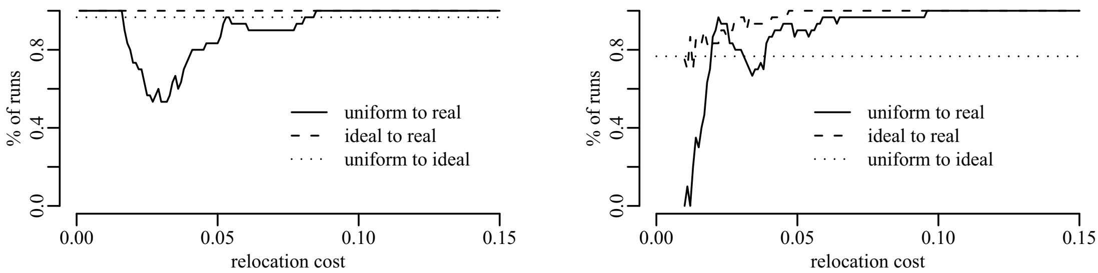

Figure 5: The share of runs where the majority of inhabitants preferred one system to another. Only the runs where an equilibrium is reached are considered. The left panel is for $a_{\min} = 0$, the right panel for $a_{\min} = 0.3$.

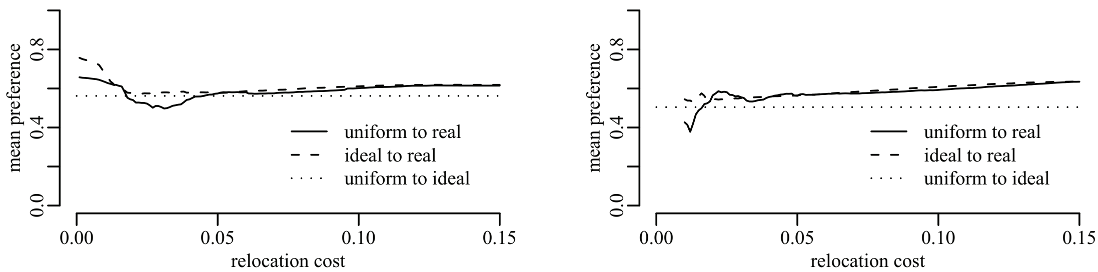

Figure 6: The mean share of inhabitants who preferred one system to another. Only the runs where an equilibrium is reached are considered. The left panel is for $a_{\min} = 0$, the right panel for $a_{\min} = 0.3$.

5 Conclusion

The simulation model shows that Tiebout [4] was overly optimistic. His conjectures hold true only in a world with inhabitants that are identical in all respects other than their "taste" for public goods. If the inhabitants differ in their productivities, the strategic interactions among them considerably modify their choice of a region. There need not to be an equilibrium if the relocation cost is too small. But even if there is an equilibrium, it does not possess the desirable features expected by Tiebout. First, the inhabitants are not located in their optimal regions. Second, the aggregate quantity of public goods is lower than optimal, and public goods and the tax burden are distributed among the inhabitants in an inefficient way. As a result, the real version of the Tiebout's system is preferred by a majority sufficient to establish it only under a rather restrictive conditions.

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