Markets, social networks, endogenous preferences, and opinion leaders[1]

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Abstract. This paper studies the impact of opinion leaders ("stars") and their fans on equilibrium market prices within the Bell's model (JEBO 2002). The simulation shows that 1) the model may not converge when the opinion leader consumes the good that is extremely scarce—it can create infinite cycles in her fans' preferences; 2) the preferences may not be completely polarized in the same situation—the agents with non-polarized preferences prevent the cycles; 3) while the agents in the Bell's model consume only the more abundant good when the other good is extremely scarce, the presence of the opinion leader eliminates this when she consumes the scarce good, and 4) the presence of the opinion leader and her fans can sometimes surprisingly lower the price of the good that the opinion leader consumes.

Keywords: endogenous preferences, market, social network, opinion leaders, agent-based simulation

JEL classification: D83, D51, D85
AMS classification: 68U20, 91D30, 91B69

1 Introduction

The seminal agent-based study on endogenous preferences and how they are influenced by interactions within a social network is A. M. Bell's paper "Locally interdependent preferences in a general equilibrium environment" [1] which explored the endogenous preference adaptation for a grid social network. The robustness of her conclusions was later tested by Kvasnička [2] who showed that some of Bell's original conclusions do not apply for other symmetric network structures. The present paper enhances the Bell's model in another way: it studies the impact of an opinion leader ("a star") on human preferences within the framework of Bells model. It introduces a new kind of agent ("an opinion leader" or "a star") and a second social network, the asymmetric star network of her fans. By means of agent-based simulations it investigates how the properties of the fans network (number of agents in the network, their concentration, and the strength of the opinion leader's impact on one fan) affect the structure of the model equilibrium and the relative price of the two goods. It is the first step toward a quantitative modeling of an important marketing problem how opinion-leaders (e.g. pop stars) can be used to enhance the demand for one of competing products in environments where fashion matters.

2 Model

The model enhances Bell's model of "exchange economy", see [1]. There are two kinds of agents: "ordinary people" and one "opinion leader". All ordinary agents consume two comparable kinds of goods (e.g. white and black t-shirts). The opinion leader consumes only good 1 (the white t-shirts). In every period, all agents get an initial endowment of each good which they exchange with each other in the centralized market at the market clearing price. The ordinary agents' preferences evolve over time: each ordinary agent increases her preference for the good that has been recently more popular (i.e. more consumed) in her neighborhood. Her neighborhood consists of three parts: 1) the agent herself, 2) her eight closest agents, and 3) the opinion leader, if the agent is her fan. The opinion leader's preferences do not change in time.

More formally, there are N agents: one opinion leader indexed $i = 1$ and $N - 1$ ordinary agents indexed $i = 2, \ldots, N$. In every period, each agent gets the same endowment: e_1 units of good 1 and e_2 units of good 2 ($e_1, e_2 > 0$,

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and $e_1 + e_2 = 100$). Agent i then demands x_{i1} units of good 1 and x_{i2} units of good 2 to maximize her one-period Cobb-Douglas utility function subject to the constraint given by her endowment, i.e.

$$\max_{x_{i1},x_{i2}} x_{i1}^{a_{it}} x_{i2}^{1-a_{it}} \qquad \text{s.t.} \quad p_1 x_{i1} + p_2 x_{i2} = p_1 e_1 + p_2 e_2, \tag{1}$$

where p_1 and p_2 are the prices of good 1 and good 2 respectively, and a_{it} is agent i's relative preference for good 1 at time t. The initial value of the preference parameter a_{i0} is drawn independently for each ordinary agent $i > 1$ from the continuous uniform distribution $U(0, 1)$; the opinion leader's preference $a_{1t} = 1$ for each t.

Agent i's demand for the two goods is then

$$x_{i1}(p_1, p_2) = a_{it}\left(e_1 + \frac{p_2}{p_1}e_2\right), \qquad x_{i2}(p_1, p_2) = (1 - a_{it})\frac{p_1}{p_2}\left(e_1 + \frac{p_2}{p_1}e_2\right). \tag{2}$$

Since the total endowment is given, the market clearing relative price of good 1 in terms of good 2 is

$$\frac{p_1^*}{p_2^*} = \frac{e_2 \sum_{\forall j} a_{jt}}{e_1 \sum_{\forall j}(1 - a_{jt})}. \tag{3}$$

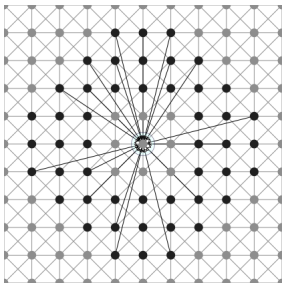
The relationships among the agents are defined through two kinds of social networks: one network defining friends, and the other defining fans of the opinion leader. Each social network is represented by a graph, in which agents are vertices and their relationships are edges (connections). Agent i has relationship to agent j in graph G if i is connected with an edge to j; we then write $i \sim j \in G$. Both social networks are created independently for each simulation and are fixed within the simulation. For an example see Figure 1.

The first social network portrays friendship, and is equivalent to the Bell's original social network, see [1]. It is represented by an undirected graph G_1, which is a grid network on a toroid; see [6] for its definition. (Since the graph G_1 is undirected, the friendship relationship is symmetric, i.e. $i \sim j \in G_1 \Leftrightarrow j \sim i \in G_1$.) One can see the network as a cellular automaton on a lattice where each agent is represented by a cell, the set of the agent's friends consists of the cell's Moore neighborhood, and the lattice edges are connected to each other in such a way that an agent at the very top of the lattice is at the same time located also at the very bottom of the lattice, and so on. Let us define index function $n(i) = \{j : j = i \vee j \sim i \in G_1\}$ as the set of indices of agent i's friends and the index of the agent i herself.

The second social network portrays the relationships of the fans to the opinion leader. This network is represented by a directed star graph G_2; see [6] for its definition. In this network, the fans are connected to the opinion leader in such a way that each fan cares about the consumption of the opinion leader but not about the consumption of the other fans; the opinion leader does not care about the consumption of her fans. (The relation in directed networks is neither reflexive, nor symmetric, i.e. $1 \sim 1 \notin G_2$, and $i \sim j \in G_2$ does not imply $j \sim i \in G_2$.) The fans are selected from among a subset A of the ordinary agents that are not the opinion leader's friends. Let $\rho = |A|/(N - 9)$ denote the share of the agents that can be selected as opinion leader's fans on the total population of all ordinary agents except the opinion leader's friends. Let $\pi \leq \rho$ denote the share of the actual fans on the total population of the ordinary agents that are not the opinion leader's friends. The network G_2 is constructed this way: 1) construct the set A that contains indexes of all ordinary agents that are not the opinion leader's friends that are located within the smallest circle around the opinion leader that includes at least $\rho(N - 9) + 9$ agents (I assume that the agents are distributed equidistantly in the friendship space, as on the lattice); 2) construct the set B as $\pi(N - 9)$ randomly chosen indexes from the set A, and 3) connect the agents in B to the opinion leader in G_2, i.e. set $j \sim 1 \in G_2$ for $\forall j \in B$. Notice that the parameter ρ together with π determines how concentrated within the whole population the fans are. For instance, $\pi = 0.1$ and $\rho = 1$ means that there are few fans and they are sparsely scattered within the whole population of agents. On the other hand, $\pi = 0.3$ and $\rho = 0.3$ means that all the fans are concentrated around the opinion leader—the 30 % of opinion leader's closest non-friend agents are her fan, and there are no fans farther from the opinion leader. Let us define Boolean function f such that $f(i) = 1$ if $i \sim 1 \in G_2$, and $f(i) = 0$ otherwise.

Agent $i > 1$'s neighborhood thus consists of the agent herself, her friends (i.e. the agents with indexes in $n(i)$), and the opinion leader if the agent is her friend (i.e. $f(i) = 1$). After observing the consumption in her neighborhood, each ordinary agent $i > 1$ adjusts her preferences in such a way that she increases the preference for the good that is consumed more in her neighborhood. Specifically, agent $i > 1$ sets the future value of her preference parameter $a_{i,t+1}$ at

$$a_{i,t+1} = a_{it} + r\left(\frac{\sum_{j\in n(i)} x_{j1} + sf(i)x_{11}}{\sum_{j\in n(i)} x_{j1} + sf(i)x_{11} + \sum_{j\in n(i)} x_{j2} + sf(i)x_{12}} - 0.5\right) \quad \text{for } i > 1, \tag{4}$$



The dots denote the agents, the lines denote the edges of the social networks. The agents on the very top wrap over to the very bottom and the agents at the right edge wrap over to the left edge.
The encircled dot in the middle of the figure is the opinion leader.
The black dots are the agents that can be selected as fans, i.e. agents with indexes $i \in A$.
The short gray vertical, horizontal, and diagonal lines are the vertices of the friends network G_1.
The long black lines are the vertices of the fans network G_2.

Figure 1 An example of the social networks for $N = 100$, $\pi = 0.2$, and $\rho = 0.5$.

where the adjustment parameter $r \in (0, 1)$ regulates the speed of the preference adjustment and s is the strength of impact of the opinion leader's consumption on a fan. Setting $s > 1$ allows us to study the cases where the fans value the consumption of the opinion leader higher then the consumption of their friends.

The evolution of agents' preferences and consumption is simulated in the agent-based computational fashion (for introduction to it, see e.g. [4]). First, the model is initialized: N agents are created, assigned their random initial preference $a_{i,0}$ drawn from $U(0, 1)$, and organized in the social networks G_1; then one agent is selected as the opinion leader and her preference is changed to $a_{1,0} = 1$; then the social network G_2 is created. The simulation than proceeds in discrete steps repeated until the model converges (i.e. the agents' preferences change no more), or the maximal amount of steps is reached. In each step, 1) the market clearing relative price p_1^*/p_2^* is calculated for the agents' current preferences (equation 3), 2) each agent's equilibrium consumption is calculated (equation 2), and 3) each agent $i > 1$'s preference is adjusted (equation 4).

3 Results of simulations

The model has been simulated for $N = 100, 900$, and 2 500 agents, the concentration of fans $\rho = 0.1, 0.2, \ldots, 1$, the share of fans $\pi = 0.1, 0.2, \ldots, 0.7$ (only combinations with $\pi \leq \rho$ were simulated), and the strength of opinion leader's impact $s = 1, 2, 3$. The adjustment constant r was set to 0.5 as in [1]. Three variants of the model were simulated for each random seed: 1) the "grid" model, i.e. Bell's original model without the opinion leader and her fans where agent 1 was treated as an ordinary agent, 2) the "stubborn" model, i.e. an intermediate model where agent 1 had the preference parameter $a_{1t} = 1$ at each t but had no fans (i.e. G_2 was not present), and 3) the "leader" model, i.e. the full model described above. The three variants can be straightforwardly compared because only the initialization is stochastic (the rest of the simulation is deterministic), and the order of steps in the initialization secures that the common part of each two variants is the same for each initial random seed. The model was simulated one hundred times for each feasible combination of parameters and each variant. The total number of the simulation runs was 573 300 triples. The maximal amount of simulation steps was set to 5 000. The model was simulated in NetLogo 5.0.4 [5] and the results were analyzed in R [3]. The web interface of the model is available at http://www.econ.muni.cz/~qasar/english/models.html.

3.1 Simulation outcomes

Bell [1] discusses two kinds of simulation outcomes on the grid friends network G_1. In both, the system converges, each good is consumed by some agents, and fewer agents consume the scarcer good than the more abundant one. The difference between the outcomes lies in the agents' equilibrium preferences. In the first case, each agent specializes in consumption of only one kind of good (i.e. $a_{it} \in \{0, 1\}$, which Bell calls "polarized" preferences), while in the second case, at least some agents consume both goods (i.e. have $a_{it} \in (0, 1)$). Bell claims that the second outcome is unstable, and hence cannot occur in a simulation [1, p. 321]. In the polarized state, the agents with the same preference are clustered together (for an example of the clusters, see Figure 3, panel a). The clusters arise because no ordinary agent can keep its polarized preference unless she is surrounded by a sufficient number of other agents with the same preference. The reason why more agents specialize in consumption of the more abundant good is the negative feedback provided by the market: when the number of agents consuming a good is out of proportion to the good endowment, the relative price of the goods is unequal; the consumers of the cheaper good consume more, which motivates some consumers of the other good to change their preferences. In the limit, this market feedback presses the relative price of the goods to unity. However, there is also the "bandwagon" effect: the higher the proportion of consumers of one good, the higher the probability that an agent is surrounded

by the consumers of this good, and hence that she switches her preference to this good. The bandwagon effect rises demand for the more abundant good, and hence presses the relative price of the goods away from unity. See [2] for a more detailed analysis.

Kvasnička [2] showed that there may be a richer set of outcomes with a general social network: 1) the simulation may not converge, 2) it may converge but some agents can remain non-polarized, 3) the simulation can converge, all agents can be polarized but all of them consume only the more abundant good, and 4) the simulation converges, all agents are polarized and both kinds of good are consumed by some agents, which is the Bell's standard outcome. All these four states occurred also in the present simulation, though the first three cases occurred only with extreme proportions of the endowment: either $e_1 \geq 95$ and $e_2 \leq 5$, or $e_1 \leq 5$ and $e_2 \geq 95$. The standard logistic regression was not suitable here for the detection of the combinations of treatments for which these outcomes were realized because the model behavior is highly non-linear; decision trees constructed by R packages rpart, rpart.plot and Rattle [7] were used instead.

The state 1, the non-convergence, is caused by fans network G_2—the grid and stubborn models always converge. The model with the opinion leader did not converge in 22 737 out of 573 300 runs (i.e. about 3.97 % of runs). Most non-convergent runs happened when $e_2 = 99$ (i.e. $e_1 = 1$), $N = 900$ or 2 500, and $\pi \geq 30$ %. For details, see the decision tree in Figure 2. When the model does not converge, it is because it cycles. Let us show the mechanism on an example. Assume the model with $N = 2500$ agents, endowments $e_1 = 1$ and $e_2 = 99$, $\pi = 0.2$, $\rho = 0.3$ and $s = 2$. Because of the bandwagon effect, the model soon reaches the state where only the opinion leader consumes the scarce good 1 and all other agents consume only the abundant good 2. The opinion leader then consumes 2 500 units of good 1, while each other agent consumes only about 99 units of good 2. Thus according to equation (4), all fans increase their taste for good 1 in the next step. This rises the demand for good 1, which rises its relative price, and the consumption of all agents who consume good 1 declines. In the following step, the fans' preferences for the good drop according to equation (4) again. The opinion leader is again the only agent consuming the scarce good 1, and the new cycle begins.

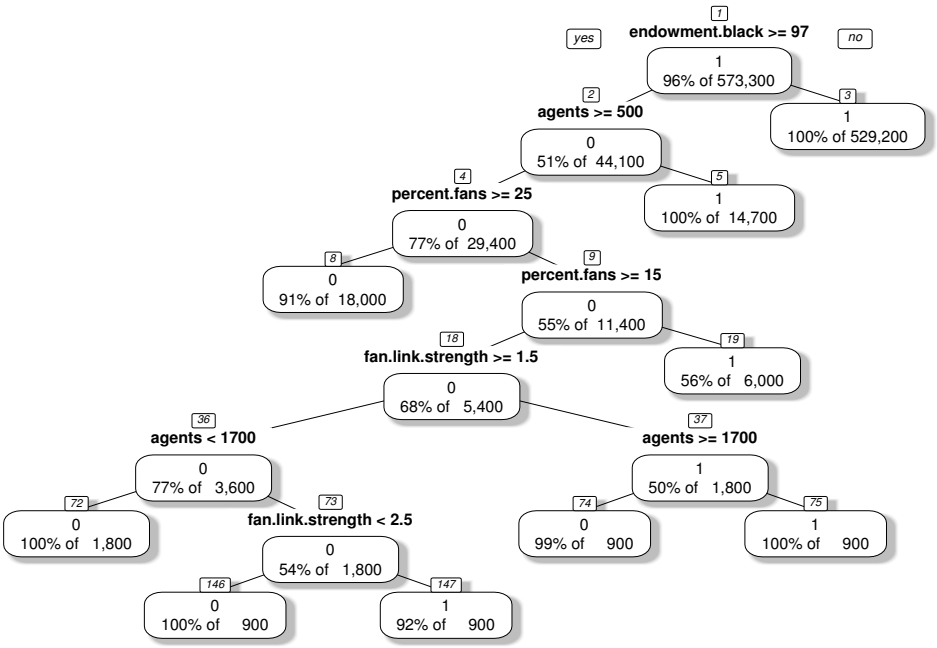


Figure 2 Decision tree showing when the leader model converged. Convergence is denoted by 1, N is denoted as agents, e_2 as endowment.black, π as percent.fans, and s as fan.link.strength.

The state 2, where the model converges but some agents have non-polarized preferences, occurs in 4 930 runs out of the 550 563 converged runs (i.e. 0.9 % of runs). It only occurs in the leader model when $N = 100$, $e_1 = 5$ and $e_2 = 95$, and $s \geq 2$. Usually, there were one to four non-polarized agents (the fans) that advanced the relative price of the scare good 1 in such a way that the opinion leader's consumption of good 1 was low enough, so that the other fans were not tempted to change their preferences. This balancing problem obviously becomes more delicate when there are more agents, and the state was not observed in data for higher N.

The state 3, where the model converges, all agents' preference are polarized but only the more abundant good is consumed because of the bandwagon effect, occurred in the grid model when the endowment was extremely unequal. It always occurred in the grid model when $e_j = 99$ (and $e_{-j} = 1$), and often also when $e_j = 95$ and $N = 100$. Good 1 was solely consumed in 31 650 and good 2 in 58 653 runs out of 545 633 converged polarized runs totally (together 16.6 % of the converged polarized runs). The presence of a stubborn consumer of good 1 (whether she had a fans network or not) eliminated this state when the scarce good was good 1—it was at least consumed by agent 1 herself. When the scarce good was good 2, then the presence of a stubborn consumer of good 1 slightly rose the number of runs when this happened: the state occurred in 58 946 runs in the stubborn model and in 59 015 runs in the leader model.

3.2 Opinion leader's impact on relative price

To study the impact of the opinion leader' presence on the relative price of good 1, we will confine to the converged polarized runs when both goods were consumed in positive quantities. It may seem intuitive that the presence of the opinion leader that always consumes good 1 and her fans that can follow her in her consumption should rise the demand for good 1, and hence its price. However, it is not this simple. In fact, the inclusion of the opinion leader and her fans increased the relative price of good 1 in 360 910 runs out of 454 703 runs in view (i.e. in 79.4 % runs), did not change it in 35 973 (7.9 %) runs, and decreased it in 57 820 (12.7 %) runs. It is instructive to decompose the total effect to the impact of the opinion leader herself (the change from the grid to the corresponding stubborn model) and the impact of the fans network (the change from the stubborn model to the corresponding leader model). The detailed decomposition is presented in Table 1.

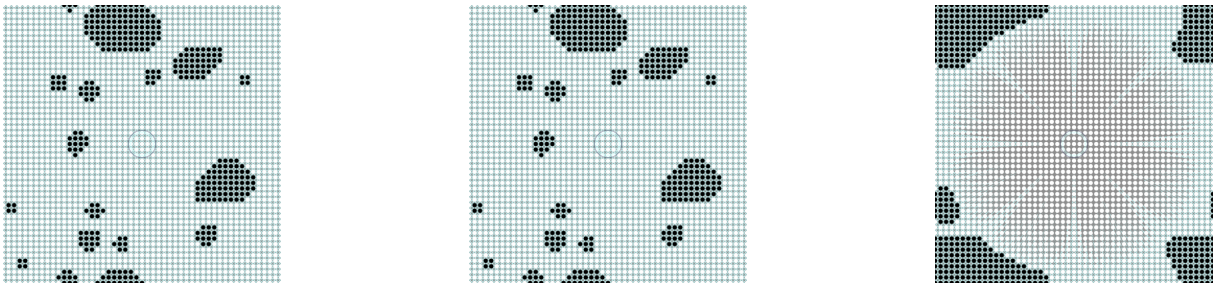
	total rise			total no change			total decline		
	decline	no change	rise	decline	no change	rise	decline	no change	rise
decline	0	0	94 343	0	0	4 988	7 842	3 298	6 189
no change	0	0	123 588	0	27 790	0	31 465	0	0
rise	10 477	8 885	123 617	3 195	0	0	9 026	0	0

Table 1 Number of converged polarized runs where both goods were consumed decomposed. Each block corresponds to a total change in the relative price; e.g. the block denoted "total rise" includes the runs where the relative price of good 1 rose with transition from the grid model to the leader model. Within the blocks, the rows correspond to a change of the relative price when one agent was assigned the stubborn preference for good 1 (the transition from the grid to the stubborn model). The columns correspond a change of the relative price when the stubborn agent was given the fans network G_2 (the transition from the stubborn to the leader model). Each number is the number of the corresponding runs; e.g. 4 988 is the number of runs where the relative price declined when the stubborn consumer of good 1 was introduced, it rose again when the stubborn agent was given a network of fans, and the two effects had precisely the same magnitude with the opposite sign, so that the relative price was the same in the leader model as in the corresponding grid model. (Note that the relative price can take only values from a discrete set because of equation (3), polarized preferences, and discrete number of agents.

When one agent in the grid network was assigned the stubborn preference for good 1 (i.e. the grid model was changed into the stubborn model), the relative price of good 1 rose in 34 % runs, did not change in 40 % runs, and decreased in 26 % runs. The reason is that when one agent in the grid network is given a stubborn preference for good 1, the clusters of consumers may evolve differently, and the number of the consumers of each good can randomly rise, decline, or stay the same (as the transition from panel a to panel b in Figure 3). The relative price of good 1 declines more often when the number of agents is high and the endowment e_2 is high. For instance, the relative price of good 1 declines in 53 % runs when one stubborn consumer of good 1 is added to the grid network of $N = 2500$ agents with endowment $e_2 = 90$, but it declines in only 9 % runs with $N = 100$ and $e_2 = 10$.

When the fans network G_2 was added (i.e. the stubborn model was changed into the leader model), the relative price of good 2 rose in 77 % runs, did not change in 9 % runs, and declined in 14 % runs. The relative price declined more often when the endowment of good 1 was higher then the endowment of good 2 (72 % of this relative price declines happened when $e_2 \leq 40$), and when the number of agents was high (80 % of the relative price declines happened when $N \geq 900$). The reason is again that the clustering evolves differently when the fans network G_2 is added. As the clusters evolve differently, the number of the consumers of each good can randomly rise, decline, or stay the same, and the relative price changes accordingly.

The most interesting case of the decline of the relative price of good 1 caused by introduction of the fans



(a) grid model (b) stubborn model (c) leader model

Figure 3 An example of the outcome of the simulation where the relative price of good 1 consumed by the opinion leader decreased when because of the network G_2 of her fans. Here $N = 2500$, $e_2 = 20$, $\pi = \rho = 0.7$, and $s = 3$. The opinion leader or corresponding agent has been moved to the center and denoted by the circle. The black dots correspond to the agents with preference $a_{it} = 0$, the white dots to the agents with preference $a_{it} = 1$.

network G_2 happens when the opinion leader's fans are sufficiently concentrated, i.e. π/ρ is close to unity. An example can be seen in Figure 3. The introduction of the fans network G_2 (the transition from the panel b to the panel c) cleanses the surrounding of the opinion leader, and creates a great cluster of consumers of good 1 which rises the demand for good 1, and hence its relative price. The side effect is that consumers of good 2 often create a smaller number of bigger clusters, which allows more agents to keep the preference for good 2. This second effect lowers the relative price of goods 1. Quite often the second effect is stronger than the former one, and the relative price of good 1 declines.

4 Conclusions

The simulation shows that the presence of an opinion leader and a network of her fans within Bell's model modifies Bell's results: 1) The model may not converge because the presence of the opinion leader that stubbornly consumes a good that is extremely scarce can create infinite cycles in her fans' preferences. 2) The preferences need not be completely polarized in the same situation—the agents with non-polarized preferences can eliminate the cycles. 3) While the agents in the Bell's model consume only the more abundant good when the other good is extremely scarce, the presence of the opinion leader can eliminate this effect when she consumes the scarce good, or slightly enhance it when she consumes the abundant good. 4) The most surprising effect is that the presence of the opinion leader and her fans can sometimes lower the demand, and hence the price of the good that the opinion leader consumes. This suggests that demand manipulation through opinion-leaders may be risky even when the leader succeeds in rising demand of her fans.

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