Monopoly Supply Chain Management via Rubinstein Bargaining

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Abstract. This paper argues that the Spengler’s (JPE, 1950) double-markup story does not capture the strategic situation faced by a monopoly supply chain. We show that 1) the Spengler’s double-markup model is equivalent to an extensive game in which the upstream monopoly has all power to set the intermediate price, and the downstream monopsony has no bargaining power. This assumption seems unrealistic. 2) We model this situation using the Rubinstein bargaining. The equilibrium of our model does not yield a double markup. Hence, the joint output, the consumers price, and the total profit is as high as those of a vertically integrated firm. Moreover, the profit is split roughly equally between the two firms. 3) Our model has different implications for the competition policy.

Keywords: vertical integration, double markup, Rubinstein bargaining, industrial organization, competition policy

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1 Introduction

The established conception of the supply chain behavior was derived from the Spengler’s model (it is the basic model of supply chain management in most books on industrial organization and competition policy, see e.g. [2, 5, 7, 8]; it is also the main model tested experimentally, see e.g. [1]). In his seminal paper [6], Spengler showed that the firms’ attempt to maximize their profit independently leads to the “double markup” if the firms constitute a supply chain whose stages operate on non-competitive markets. Such a chain produces lower quantity, charges higher price, and gains lower profit than a vertically integrated firm. The Spengler’s model was refined later for the case of a monopoly supply chain. In this case, all Spengler’s conclusions hold true. Moreover, the total profit of the chain is split highly unevenly among the firms: the upstream firms always gain more than the downstream firms. The fact that supply chain might be even less efficient than one vertically integrated monopoly has important, but ambiguous implications for the competition policy. In some market structures, vertical mergers of firms in a monopoly supply chain should be allowed, for they increase the efficiency, see [2]. In other market structures (e.g. when there are two upstream firms in Bertrand competition and two downstream firms in Cournot competition), mergers should not be allowed, see [5].

In this paper, we deal with the monopoly refinement of the model only. We argue 1) that the Spengler’s double markup story does not capture the strategic situation faced by the bilateral monopoly he envisioned. We show that the double markup model is equivalent to an extensive game in which the upstream monopoly is first on the move: it has all power to set the intermediate price, and the downstream monopsony has no bargaining power and acts as a price taker. This is the reason why the upstream firm has always higher profit than the downstream firm. This assumption seems unrealistic—the downstream firm could try to bargain for a better deal. Therefore, it would be more realistic to model the strategic situation as a bargaining. 2) Our model based on the Rubinstein bargaining has strikingly different results from the Spengler’s one. First, there is no double markup: the output, final price and the total chain’s profit are the same as those set by a vertically integrated monopoly. Second, the chain’s total profit is split between the firms based on which one is first on the move, not based on their position in the supply chain, and is split roughly evenly under fairly realistic conditions. 3) Since the monopoly supply chain is precisely as (in)efficient as the vertically integrated monopoly, the established implications for the competition policy are no
Notice that the profit of the upstream firm $\pi_u$ is the total revenue of the downstream firm on the consumer market, and $C_u(q)$ and $C_d(q)$ are the total costs of the upstream and the downstream firms respectively.

For simplicity’s sake, we assume in this section that the total cost of the upstream firm is $C_u(q) = c_u \cdot q$, the total cost of the downstream firm is $C_d(q) = c_d \cdot q$, and that the downstream firm’s total revenue $R(q) = \hat{p} \cdot q$ is given by the inverse demand for its production $\hat{p} = a - b \cdot q$, where $a > c_u + c_d$ and $b > 0$. The profits of the upstream and the downstream firms respectively are then

$$\pi_u = p \cdot q - C_u(q),$$
$$\pi_d = \hat{p} \cdot q - p \cdot q - c_d \cdot q = (a - b \cdot q) \cdot q - (p + c_d) \cdot q. \quad (3, 4)$$

The conventional solution of the Spengler’s model proceeds in three steps: 1) we derive the demand for the upstream firm’s production, 2) we calculate the intermediate price $p$ set by the upstream firm, and 3) we calculate the price and quantity set by the downstream firm. In other words, first we calculate the output that maximizes the downstream firm’s profit (4) taking the intermediate price $p$ as given. The downstream firm’s optimal output $\hat{q}^*$ is then

$$q^* = (a - p - c_d)/2b. \quad (5)$$

Since both firms produce the same quantity $q^*$, the equation (5) is the demand for the upstream firm’s production. Second, we substitute (5) into (3) and calculate the intermediate price $p^*$ that maximizes the upstream firm’s profit (3). Third, substituting the optimal upstream firm’s price $p^*$ into (5) we calculate the total chain’s output $q^*$, the downstream firm’s optimal price $\hat{p}^*$, and the profits of both firms. It holds that

$$q^* = \frac{a - c_u - c_d}{2b}, \quad p^* = \frac{a + c_u + c_d}{2}, \quad \hat{p} = \frac{3a + c_u + c_d}{4}, \quad \pi_u = \frac{(a - c_u - c_d)^2}{8b}, \quad \pi_d = \frac{(a - c_u - c_d)^2}{16b}. \quad (6)$$

Notice that the profit of the upstream firm $\pi_u$ is always higher than the profit of the downstream firm $\pi_d$; $\pi_u = 2\pi_d$ in the linear case.

We can compare the outcomes of the monopoly supply chain behavior to those of the vertically integrated monopoly, i.e. a firm created by a merger of the upstream and the downstream firms. We assume there is no economy of scale or scope and the total cost $C_{v_i}(q)$ of the vertically integrated monopoly is $C_{v_i}(q) = C_u(q) + C_d(q)$. The profit $\pi_{v_i} = \Pi_{v_i}(q)$ of the vertically integrated monopoly is

$$\pi_{v_i} = \Pi_{v_i}(q) = R(q) - C_{v_i}(q) = R(q) - C_u(q) - C_d(q). \quad (6)$$

The vertically integrated monopoly with the linear demand $p = a - b \cdot q$ and the total cost $C_{v_i}(q) = (c_u + c_d) \cdot q$ would produce the quantity $q'' = (a - c_u - c_d)/(2b)$, sell it for the price $p'' = (a + c_u + c_d)/2$, and reach the profit $\pi_{v_i} = (a - c_u - c_d)^2/(4b)$. It is easy to see that the monopoly output $q''$ is higher than the chain’s output $q^*$ ($q'' = 2q^*$ in the linear case), the monopoly final price $p''$ is lower than the chain’s final price $\hat{p}^*$, and the monopoly profit $\pi_{v_i}$ is higher than the chain’s total profit $\pi_u + \pi_d$. Thus in terms of welfare the chain’s output is inferior to the output of the vertically integrated monopoly since both the consumers’ and producers’ surpluses are higher in case of the vertically integrated monopoly.
The conventional solution may seem straightforward but it is tricky. It is equivalent to the backward induction used to solve extensive games. In fact, the double markup model is a two stage extensive game. There are two players in this game: the upstream and the downstream firm. The strategy of the upstream firm is to set the intermediate price \( p \geq 0 \), the strategy of the downstream firm is to set the chain’s output \( q \geq 0 \). The upstream firm is first on the move: it sets the intermediate price \( p \). The downstream firm is on the move in the second stage of the game, where it takes the intermediate price \( p \) as given, and sets the chain’s quantity \( q \). The payoffs of the game are given by the profits \( \pi_u \) and \( \pi_d \), i.e. equations (3) and (4), respectively. The game is solved by the backward induction. First, we solve the second stage of the game—the subgame where the downstream firm is on the move. We find its best response to a given intermediate price \( p \). This best response is the demand for the production of the upstream firm (5). Second, we calculate the optimal behavior of the upstream firm given the downstream firm’s best response function. Third, we solve for the remaining variables. It is evident that the particular steps of the conventional solution correspond precisely to the steps of the backward induction, and hence the models are equivalent. The fact that the double markup model is equivalent to this extensive game also explains why the upstream firm’s profit is always higher then the downstream firm’s profit—it is advantageous to be first on the move in this game because \( \hat{p}^2 = (a + c_d + p^0)/2 \). Thus if the upstream firm rises its price \( p \) by 1, the downstream firm rises its price \( \hat{p} \) only by 1/2, i.e. the downstream firm accommodates a part of the price increase. The upstream firm then has a strong incentive to increase its price \( p \) as long as its markup rises faster that the quantity demanded decreases.

The extensive game presented above does not grasp the strategic situation faced by the two firms in the chain properly. In the double markup model, the upstream monopoly has a full monopoly power to set the intermediate price \( p \), while the downstream firm (which is really a monopsony) acts as a price taker. There is no reason why a monopoly should accept such a passive role, especially when the resulting split of the total chain’s profit is unfavorable to it. It seems more likely that the downstream firm would try to bargain for a better deal. Moreover, it is not obvious why the upstream firm should be first on the move and why any firm would like to play the double-markup game at all. There are many other extensive games the firms could play. For instance, any firm might try to play “take it or leave it” game. If the upstream firm was first on the move, it could ask the downstream firm to sell the monopoly quantity \( q^m \) for the monopoly price \( p^0 \) while charging \( p = p^0 - c_d \). Since the downstream firm’s payoff is zero regardless of whether it accepts and rejects the offer, it would accept it, and the upstream firm would gain full monopoly profit \( \pi_u \). On the other hand, if the downstream firm was first on the move, it could ask the upstream firm to sell it the monopoly quantity \( q^m \) for the upstream firm’s marginal cost \( c_u \), and hence gain the whole monopoly profit \( \pi_u \) itself. There are many other extensive games the firms could try to play. However, all of them are susceptible to the same criticism: the other firm has no reason to accept the game and would most likely try to bargain for a better deal. This is why we believe that the strategic situation faced by the bilateral monopoly of the firms in the monopoly supply chain is best modeled by a model of bargaining.

### 3 Supply Chain Management via Rubinstein Bargaining

In this section we provide a model of a bargaining process between the upstream and the downstream firm. The model is based on the Rubinstein bargaining game [4] where players take turns to make offers to each other until agreement is reached. In the Rubinstein game players bargain over the division of fixed size pie. In our model, bargaining determines not only the division of the pie but also the size of the pie. Consider an industry described in section 2 where profits of the upstream firm and the downstream firm are given by equations (1) and (2) respectively. We assume that there is a unique quantity \( q^m > 0 \) that maximizes the profit of vertically integrated monopoly \( R(q) - C_u(q) - C_d(q) \), but we do not impose any other restrictions on the revenue function or cost functions.

We define the bargaining game with alternating offers between the upstream firm and the downstream firm as follows: Firms bargain over the intermediate price \( p \) and production level \( q \) sold by the downstream firm to final consumers. (Alternatively, the firms can also bargain over the intermediate price \( p \) and the price \( \hat{p} \) at which the downstream firm sells the product to the final consumers.) Time is discrete. At time 0 one of the firms, for example the upstream firm, offers to the downstream firm a price \( p_u \) and production quantity \( q_u \). If the downstream firm accepts the offer, then the agreement is reached. On the other hand, if the downstream firm rejects the offer, then it makes an counteroffer \( (q_d, p_d) \) in the next period. The upstream firm can again accept or reject the offer. This process of making offers and counteroffers continues until the agreement is reached. Preferences of the upstream firm and the downstream firm are given by their discounted profits \( \delta_u \Pi_u(q, p) \) and \( \delta_d \Pi_d(q, p) \), where \( \delta_u \in (0, 1) \) denotes the discount factor of the upstream firm, \( \delta_d \in (0, 1) \) denotes the discount factor of the downstream firm and \( T \) is the period when the agreement is reached.
Note that since the game has infinite horizon, we cannot use backward induction method. We proceed as follows. First, we find strategy profile that satisfies two properties stated below and constitutes a reasonable candidate for the subgame perfect equilibrium (SPE). Next, we prove that one-deviation property holds for this SPE. Finally, we prove that this SPE is unique. We assume that the game has equilibrium in stationary strategies. The stationary structure of the game does not necessarily imply that the game has equilibrium in stationary strategies, the stationarity of strategies provides a reasonable starting point. These two assumptions imply that whenever a player makes an equilibrium offer, the offer is accepted.

Consider a subgame in which the upstream firm is making an offer. It follows from the two above stated assumptions that the downstream firm’s profit from rejecting the offer is \( \delta_d \Pi_d(q^*, p^*_d) \) where \((q^*, p^*_d)\) is the equilibrium offer made by the downstream firm. The SPE requires that the downstream firm accepts every offer \((q_a, p_a)\) such that \( \Pi_d(q_a, p_a) \geq \delta_d \Pi_d(q^*, p^*_d) \). This condition will be binding, because if \( \Pi_d(q_a, p_a) > \delta_d \Pi_d(q^*, p^*_d) \), then the upstream firm can increase its profit by offering a slightly higher price. The upstream firm is thus solving the following problem

\[
\max_{p_a, q_a} p_a q_a - C_d(q_a) \quad \text{s. t.} \quad R(q_a) - p_a q_a - C_d(q_a) = \delta_d (R(q_a) - p_a q_d - C_d(q_d)).
\]

The solution of this problem is given by the following equations

\[
R'(q_a) = C'_d(q_a) + C'_d(q_a),
\]
\[
R(q_a) - p_a q_a - C_d(q_a) = \delta_d (R(q_a) - p_a q_d - C_d(q_d)).
\]

By a similar argument we can find that the downstream firm is solving the following problem when making an offer

\[
\max_{p_d, q_d} R(q_d) - p_d q_d - C_d(q_d) \quad \text{s. t.} \quad p_d q_d - C_d(q_d) = \delta_d (p_a q_a - C_d(q_a)).
\]

The solution of this problem is again given by two equations

\[
R'(q_d) = C'_d(q_d) + C'_d(q_d),
\]
\[
p_d q_d - C_d(q_d) = \delta_d (p_a q_a - C_d(q_a)).
\]

As we can see both firms offer the same quantity of production \(q^*\) which is the unique solution to equation (7) and (9). The level of production is such that marginal revenue equals to the sum of marginal costs. So, it is the same production level that produces vertically integrated monopoly, i.e. \(q^* = q^m\). Consequently, total profit of both firms is also the same as the profit of vertically integrated monopoly \(\Pi\). It is given by \(q^*\) and two following equations hold

\[
\Pi_u(q^*, p^*_u) + \Pi_d(q^*, p^*_d) = R(q^*) - C_d(q^*) - C_d(q^*) = \Pi,
\]
\[
\Pi_u(q^*, p^*_u) + \Pi_d(q^*, p^*_d) = R(q^*) - C_u(q^*) - C_d(q^*) = \Pi.
\]

The equilibrium price is then determined by equations (8), (10), (11) and (12). These equations have a unique solution

\[
p_u^* = \frac{1 - \delta_d}{1 - \delta_a \delta_d} \frac{R(q^*) - C_d(q^*)}{q^*} + \frac{\delta_d (1 - \delta_a) C_u(q^*)}{1 - \delta_a \delta_d} q^*.
\]
\[
p_d^* = \frac{\delta_d (1 - \delta_d) R(q^*) - C_d(q^*)}{q^*} + \frac{1 - \delta_d}{1 - \delta_a \delta_d} C_u(q^*).
\]

By substituting the equilibrium quantity and the equilibrium prices into the profit functions we can easily calculate the profit of the downstream and the upstream firm.

\[
\Pi_u(q^*, p_u^*) = \frac{1 - \delta_d}{1 - \delta_a \delta_d} \Pi, \quad \Pi_d(q^*, p_d^*) = \frac{\delta_d (1 - \delta_d)}{1 - \delta_a \delta_d} \Pi,
\]
\[
\Pi_u(q^*, p_d^*) = \frac{\delta_d (1 - \delta_d) C_u(q^*)}{1 - \delta_a \delta_d} \Pi, \quad \Pi_d(q^*, p_d^*) = \frac{1 - \delta_d}{1 - \delta_a \delta_d} \Pi.
\]

Now we have a reasonable guess of an equilibrium strategy profile. In the proof of following proposition we verify that this strategy profile constitutes a SPE. Moreover, the proposition 2 rules out the possibility of existence of another equilibria.
Proposition 1. The following pair of strategies constitutes a SPE of the bargaining game with alternating offers between the upstream firm and the downstream firm

- the downstream firm always offers \( q' \) and \( p'_d \) and accepts an offer \((q_u, p_u)\) if and only if \( \Pi_u(q_u, p_u) \geq \Pi_u(q', p'_d) \).
- the upstream firm always offers \( q' \) and \( p'_u \) and accepts an offer \((q_d, p_d)\) if and only if \( \Pi_d(q_d, p_d) \geq \Pi_d(q', p'_u) \),

where \( p'_d, p'_u \) are given by (13) and (14) and \( q' \) is a unique solution to (7) or (9).

**Proof.** We have to show that one-deviation property holds, i.e. no player can profitably deviate from equilibrium strategy at the start of any subgame. The game has four types of subgames. We show that the upstream firm’s strategy is optimal in every point in the game given the downstream firm’s strategy. By a symmetric argument, it follows that the downstream firm’s strategy is optimal in every point in the game given the upstream firm’s strategy.

1. Consider first the subgame in which the first move is the upstream firm’s offer. Suppose that the upstream firm makes an offer \((q_u, p_u) \neq (q', p'_d)\). Obviously it is not profitable to offer any \((q_u, p_u)\) such that \( \Pi_u(q_u, p_u) \leq \Pi_u(q', p'_d) \). So suppose that \( \Pi_u(q_u, p_u) > \Pi_u(q', p'_d) \). In this case the downstream firm rejects the offer and the upstream firm’s profit in the next period is \( \delta \Pi_u(q', p'_d) \). But from equations (15) and (16) we can see that \( \delta \Pi_u(q', p'_d) < \Pi_u(q', p'_d) \) and thus the deviation is not profitable.

2. Now consider the subgame in which the first move is the upstream firm’s response to the downstream firm’s offer \((q_d, p_d)\). First, assume that \( \Pi_u(q_d, p_d) \geq \Pi_u(q', p'_d) \) and the upstream firm is supposed to accept the offer. If the upstream firm rejects the offer it obtains next period payoff \( \delta \Pi_u(q', p'_d) \). But from the equations (15) and (16) we can see that \( \delta \Pi_u(q', p'_d) = \Pi_u(q', p'_d) \) and thus the deviation is not profitable. Second, assume that \( \Pi_u(q_d, p_d) < \Pi_u(q', p'_d) \) and the downstream firm is supposed to reject the offer. The rejection is clearly optimal. If the upstream firm rejects the offer it obtains next period payoff \( \delta \Pi_u(q', p'_d) \) such that \( \delta \Pi_u(q', p'_d) = \Pi_u(q', p'_d) \) and consequently \( \delta \Pi_u(q', p'_d) < \Pi_u(q', p'_d) \). Hence, the deviation is not profitable in this subgame.

\[\square\]

Proposition 2. The pair of strategies described in proposition 1 is the unique SPE of the bargaining game with alternating offers between the upstream firm and the downstream firm.

**Proof.** The proof is based on the original Rubinstein’s proof [4]. Rubinstein assumes that the size of the pie is fixed. To convert our problem into Rubinstein bargaining, we have to show that optimal production level \( q^* \) is offered at the beginning of every subgame, and hence the total profit of both firms \( \Pi(q_d, p_d) = \Pi_u(q_u, p_u) + \Pi_d(q_d, p_d) = \Pi \) is fixed. Consider a subgame starting with the upstream firm’s offer. Assume by contradiction that there is a SPE in which upstream firm offers \( p_u \) and \( q_u \neq q^* \). There are three possible endings of the game. First, suppose that the downstream firm accept the offer immediately. The downstream firm obtains payoff \( \Pi_u(q_u, p_u) \). In this case the upstream firm can offer \( q' \) and \( p'_d \) such that \( \Pi_u(q', p'_d) = \Pi_u(q_u, p_u) \). Because the downstream firm’s strategy is subgame perfect, the downstream firm accepts this offer. Moreover, we know that \( \Pi(q', p'_d) > \Pi(q_u, p_u) \) which implies that \( \Pi_u(q', p'_d) > \Pi_u(q_u, p_u) \). Hence, we have a contradiction with the assumption of SPE. Second, suppose that the downstream firm rejects the offer and the agreement is made in future period \( T \). The downstream firm obtains payoff \( \delta_{-1} \Pi_u(q', p'_d) \) and the upstream firm obtains payoff \( \delta_{-1} \Pi_d(q', p'_d) \). It does not matter who makes the offer \((q', p'_d)\). In this case the upstream firm can offer \( q' \) and \( p'_u \) such that \( \delta_{-1} \Pi_u(q', p'_d) = \Pi_u(q', p'_d) \). The downstream firm accepts this offer, because its strategy is subgame perfect. From the fact that \( \Pi(q', p'_d) \geq \Pi(q'_d, p'_u) \) and \( \delta_{-1} < 1 \) follows that \( \Pi_u(q', p'_d) > \delta_{-1} \Pi_u(q'_d, p'_u) \) which is a contradiction with the assumption of SPE. Third, consider that the agreement is never reached. But this is clearly contradiction with subgame perfection.

Similar argument holds when the downstream firm offers. Hence, we know that in every SPE each firm offers \( q^* \). From now we can suppose that the firms bargain only over the price \( p \). The price \( p \) divides the fixed size pie given by the production level \( q^* \). The problem faced by the upstream firm and the downstream firm is now the same as in the Rubinstein’s paper. So, the rest of the proof is the same as the Rubinstein’s proof of conclusion 2, see [4, p. 110].

\[\square\]
4 Conclusion

In conclusion, we would like to emphasize some important features of our bargaining model. The bargaining process is efficient in two ways. First, the agreement is reached immediately, no resources are wasted in delay. Second, the firms always supply the production level that maximizes their total profit. So, they behave just like a vertically integrated monopoly and double markup does not occur. As we can see from equations (13) and (14), the equilibrium price is a convex combination of average revenue and average cost. It is unrelated to the marginal costs or marginal revenues and it only divides the profit between the upstream and the downstream firm. In contrast with the double markup model, the division of profits is not determined by a doubtful assumption that the upstream firm moves first and sets the intermediate price. The division of profits in our model depends on the patience of firms and their starting positions. Given the patience of one firm, the other firm’s profit increases as it becomes more patient. Equations (15) and (16) imply that for any given value of \( \delta_u \), the equilibrium profit of the downstream firm increases as \( \delta_d \) increases. Symmetrically, fixing \( \delta_d \) the equilibrium profit of the upstream firm increases as \( \delta_u \) increases. The firm that starts bargaining obtains greater share of profit. If \( \delta_u = \delta_d = \delta \) then the equilibrium profit of a first-mover is given by \( \frac{1}{1+\delta} \Pi \) whereas the profit of a second-mover is \( \frac{\delta}{1+\delta} \Pi \). But the first-mover advantage disappears if \( \delta \) of both players goes to 1. In this case each firm gets one half of the profit of a vertically integrated monopoly.

References